Students' addition of decimal fractions:

The effects of context

by

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DECLARATION

I certify that this Dissertation contains no material which has been accepted for a degree or diploma by the University or any other institution, except by way of background information and duly acknowledged in the Dissertation, and to the best of the candidate's knowledge and belief no material previously published or written by another person except where due acknowledgement is made in the text of the Dissertation.

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ABSTRACT

Queensland and national curriculum documents of recent years suggest that addition and subtraction of decimal fractions can occur in a money context earlier than they do without a context. However, there does not appear to be enough research to support the legitimacy of this approach and the lack of fine detail in the curriculum documents has resulted in a variety of interpretations taken by textbook writers and presumably teachers.

Some research has shown that the use of contexts to make mathematics more relevant to students can have unintended results. Among these is the negative impact that using money problems as exemplars can have on the conceptualisation of decimal fractions. Given this finding, together with the limited guidance in the relevant curriculum documents and the variety of presentations by textbook publishers, this study sought data on the following questions:

1. How does accuracy with addition differ when decimal fractions to hundredths are written with dollar signs compared to when they are not?
2. How do addition methods differ when decimal fraction problems using hundredths are contextualised with dollar signs compared to when they are not?

A cross-sectional study was undertaken to provide a snapshot of school students' ability to undertake decimal computation addition problems in contextualised and non-context situations. Students in Years 4 and 5 in Queensland state schools completed one of two test papers. One paper presented addition problems involving decimal fractions without any context (e.g., 1.30 + 1.20). The other paper had identical questions but with a dollar sign included for each decimal fraction ($1.30 + $1.20). Altogether 161 students participated.

The results showed that there was a difference in accuracy in favour of the group working with non-contextualised decimal fractions. It was also revealed that the group working with the money context reported answers for particular questions in ways that may indicate underlying conceptual errors about money or about the relationship between money and decimal fractions. It was found that the students working without a money context preferred showing their thinking using a standard written method in greater numbers than did the students working with the contextualised problems. The latter group, in contrast, had a greater incidence of writing answers but without recording a method. Although no difference in accuracy was observed between males and females, some difference in method choice was recorded.

Keywords: addition; context; decimal fraction; primary students

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CHAPTER 1: INTRODUCTION

This study considered the responses of Year 4 and Year 5 school students in Queensland Australia to questions involving the addition of decimal fractions. Specifically it sought to determine whether contextualising the problems by including a dollar symbol made any difference to the accuracy and the nature of students' answers.

Background

Queensland is one of the eight states and territories that constitute Australia. Compulsory education in Queensland begins in Year 1 when students turn six years old before June 30, although there is a non-compulsory Prep year which is the year level before turning six (Department of Education, Training and Employment [DETE], 2012). Primary school education currently extends from Prep to Year 7, although recent changes will see Year 6 become the final year of primary school, commencing in 2015 (DETE, 2011).

Queensland schools have undergone substantial curriculum change in the past eight years. A new syllabus was introduced in 2004 but was superseded about three years later by a new document centred on "Essential Learnings and Standards" (Queensland Studies Authority [QSA], 2004, 2008). In 2011, the new national Australian Curriculum: Mathematics was released in its final form (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2011). Implementation of the Australian Curriculum in Queensland was scheduled to begin in 2012 (QSA, 2010a).

All three curriculum documents suggest that operations with decimal fractions in a money context should be addressed up to a year earlier than operations with decimal fractions that do not have any context. There currently appears to be very little research, however, on whether this advice is appropriate. Only limited guidance is given in the documents as to the type and extent of operations that should be carried out with money.

Textbook writers have interpreted the curriculum documents in various ways. Some interpretations create purely symbolic contexts for decimal fractions (e.g., simply including a dollar symbol), while others try to create context with pictures of items and money. Given the central role that curriculum documents are meant to play in deciding content for classrooms, and acknowledging the role that textbooks can play in this process, more information is needed about operations with money and their relationships to decimal fractions generally.

Some researchers (e.g., Steinle, Stacey & Chambers, 2006) have found that money may be a distraction in developing a generalised concept of decimal fractions. If it is a diversion then it is possible that encouraging operations with money may be creating problems for a generalised concept of addition with decimal fractions.

Purpose and Significance of the Study

The purpose of the study was to compare Year 4 and Year 5 students' accuracy with addition of decimal fractions when the fractions are shown either with a dollar symbol ($) or without one. The methods students used are also considered to see if context influences how solutions are recorded.

The two questions that guided the research are:

1. How does accuracy with addition differ when decimal fractions to hundredths are written with dollar signs compared to when they are not?
2. How do addition methods differ when decimal fraction problems using hundredths are contextualised with dollar signs compared to when they are not?

The results of the research could provide some guidance as to whether the currently suggested teaching sequence of operations with decimal fraction is appropriate. For example, if students working with non-contextualised problems perform as well or better than students working with problems in a money context, perhaps decimal fraction operations in their abstract form could be taught earlier. On the other hand, if students display much greater accuracy with money than for non-contextualised problems then it may be that greater emphasis needs to be placed on explaining the relationship between money and decimal fractions generally. The study may also indicate whether trying to contextualise problems with a dollar symbol is enough to create a difference in accuracy or choice of methods. The study may also shed light on the extent to which non-numerical symbols such as dollar signs are ignored, misused or misplaced when the focus is on an operation.

Definition of Terms

In this study the following terms are used.

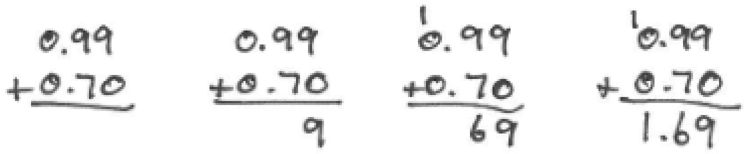
Addend. One of the numbers to be added in an addition expression (Weisstein, 2012). For example, both 2 and 3 are addends in the expression "2 + 3".

Contextualised problems and non-contextualised problems. In this study, a contextualised problem is one where an expression has been placed in the context of money by including a dollar sign ($), for example $1.30 + $1.20. The abbreviation "CX" is used as an abbreviation for "contextualised problems" including the dollar sign ($). A non-contextualised problem is one where no extra contextual information is provided, for example, 1.30 + 1.20. For these problems the abbreviation "NCX" is used.

Decimal fraction. There does not appear to be consensus on whether "decimal" or "decimal fraction" should be the preferred term as both are used in various Australian educational jurisdictions and research papers. In Queensland, the term "decimal fraction" is used in the 2004 mathematics syllabus (QSA, 2004) and Essential Learnings and Standards (QSA, 2008). Because this study took place in Queensland where it is widely used, the term decimal fraction is used in this study. A decimal fraction is a fraction whose denominator is some power of ten, usually indicated by a dot (the decimal point) written before the numerator, as $0.4 = {}^{4}/_{10}$ ("Decimal fraction", 2005).

Error. In this study, the types of responses students made that were not fully correct were classified as "major errors" or "minor errors". A major error was something that resulted in an answer that was substantially incorrect, such as recording the correct digits in the wrong place value column. A minor error was an apparent oversight or misunderstanding that resulted in an answer that was substantially correct, such as missing the decimal point in the method answer, but recording the decimal point in the reported answer.

Written methods. A standard written method is one commonly used in Australian schools for addition with any whole number or decimal fraction. It is also known as the "standard (or formal) written algorithm" (McIntosh, 2005a, p. 5). Two versions used in Australia are known as "decomposition" and "equal addends" or "equal addition" (Board of Studies New South Wales, 2002, p. 50; McIntosh, 2005a, p. 5). There are slight variations in each method but mainly in how and where "carry digits" are recorded. The decomposition method is shown in Figure 1, adapted from earlier Queensland syllabus materials, with each step in the process shown (Department of Education Queensland [DEQ], 1991). In the analysis of student methods this was the only standard written method that was used.



(a) (b) (c) (d)

Figure 1. Four steps of a standard written method to find the total of 0.99 + 0.70.

An alternative written method is any written method other than the standard written method. They are also known as informal written methods (McIntosh, 2005a). Some examples are shown in Figure 2.

(a) (b)

Figure 2. Two alternative written methods to calculate the total of 0.99 + 0.70.

An issue regarding written methods was the consideration of what an algorithm is. McIntosh (2005a, p. 5) defined an algorithm as “a set routine or procedure for performing a calculation,” and stated that there were algorithms for written and mental computation. He argued that there are standard written algorithms but also alternative written methods that if used regularly may become algorithms (i.e. a set routine). Elsewhere, McIntosh (2005b) also allowed mental methods to include jottings of sub-steps in the computation process. Under these definitions it could be argued that an algorithm may be written or mental, but the point at which a method becomes an algorithm is only going to be identified by the person using it. For the purpose of the study it was not important whether students were using a method as an algorithm but whether they were using a written method at all and, if so, which type. Thus, the term “standard written method” has been used.

Summary

The study is reported in five chapters. This chapter has described the background and significance of the study and has set up the research questions. In the next chapter, a number of background issues in relation to decimal fractions and money are examined including the statutory curriculum requirements and how these are interpreted in textbooks. The reasons for and against contextualising problems are explored, along with conceptual misunderstandings of decimal fractions, how money contexts may contribute to this confusion and what factors may affect decimal fractions operations. Chapter 3 details the study design and methodology that was used. Chapter 4 describes the results of the study and Chapter 5 discusses the findings.

CHAPTER 2: LITERATURE REVIEW

In Chapter 1 it was noted that statutory curriculum requirements indicated that operations with money should be taught before operations with decimal fractions without context. Chapter 2 provides background material on research into decimal fractions and issues around contextualising mathematical problems. It identifies statements from Queensland curriculum documents relevant to money and decimal fractions generally and considers how these statements have been interpreted by textbook developers.

Decimal Fraction Concepts

Considerable research on decimal fractions over the past few decades has focussed on conceptual misunderstandings. A number of studies and reviews have been undertaken, detailing the errors students encounter in identifying and comparing the magnitude of decimal fractions (e.g., Irwin, 2001; Isotani, McLaren & Altman, 2010; Steinle, Stacey & Chambers, 2006). Some research points to the role that everyday knowledge plays in the misconceptions. For example, Brekke (1996, p.138) noted that:

> pupils have experienced decimal numbers in connection with measurement of different kinds long before such numbers become part of instruction in school… in almost all such applications of measurement, the decimal point can be regarded as a separator between different units of measure.

The examples cited include length, mass and money. Steinle (2004) found that because measurement contexts have units and sub-units with distinct names (e.g. dollars and cents, kilograms and grams) it was easier for students to misconstrue the relationship between what is on either side of the decimal point. Thus, something such as $2.35 can be seen as two whole-number units, with two *dollars* on the left of the decimal point and thirty-five *cents* on the right of it. Further, Steinle and Stacey (1998) found that when money was reported to thousandths of a dollar (e.g. $4.993) any digit after the hundredths place was sometimes ignored, as it was interpreted as being effectively "nothing".

Addition of Decimal Fractions

Despite the readily available literature on how decimal fractions are conceptualised, there appears to very little recent research to indicate how primary school students perform with addition problems involving a wide range of decimal fractions, particularly involving hundredths. In Irwin and Britt's (2004) study of more than 1000 students, only one question on their test papers involved addition of decimal fractions and that was to tenths. Watson, Kelly and Callingham (2004) analysed more than 5000 papers from mental computation tests but only five questions addressed addition of decimal fractions and these also only involved tenths. In an evaluation of more than 900 students on the effectiveness of a particular intervention, Reys, Trafton, Reys and Zawojewski (1984) used a test that included four questions on mental addition of decimal fractions but only one question involved hundredths.

Developmental scales have been created for problems solved using mental computation (Callingham & Watson, 2004; McIntosh, 2005b) but very few of the items on the scales relate to addition of decimal hundredths. The Trends in International Mathematics and Science Study (TIMSS) for Year 4 in the years 1995, 2003 and 2007, and the National Assessment Program – Literacy and Numeracy (NAPLAN) Year 5 tests for the years 2008-2011 also provide scant evidence of assessment with addition of decimal fractions (Foy & Olson, 2009; ACARA (2010b). Tests of this type typically have few problems involving addition of decimal fractions and rarely include addition to hundredths.

There is also little that describes the errors that are specific to this type of addition. Books that address the types of errors that students make with computation, such as those by Ashlock (2010) and Hansen (2008), have some content that deals with common mistakes with decimal fraction addition, although the coverage is sparse.

One error that appears to be related to the whole-number thinking described in Section 1 is what Ashlock (2010, p. 86) called "Error Pattern A-D-1", and what Steinle et al. (2006) described as "column overflow thinking". In this type of error, each place is treated as having no relation to the places on either side of it, so that if the total for a place value column exceeds nine the total is recorded in that same column. An example is shown in Figure 3 below, where the total of 9 tenths and 7 tenths has been recorded as 16 tenths but all in the tenths column.

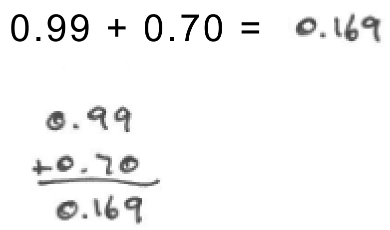


Figure 3. An example of column overflow thinking.

Among the whole number errors identified by Ashlock (2010) there are two that relate to what have been termed "ragged" decimal fractions (Benz, 1958, p. 149; Roche, 2005, p. 13; Wearne & Hiebert, 1988, p. 376). These are sets of two or more decimal fractions that have unequal numbers of decimal places, such as "0.25", "0.314" and "0.6". Adding such sets of numbers can reveal errors that Ashlock (2010, pp. 27-30) called "Error Pattern A-W-4" and "Error Pattern A-W-5". In A-W-4 with whole numbers, this error appears when adding numbers such as 23 and 6. Each digit is treated as a single digit and all are simply added together as 2 + 3 + 6. In A-W-5, adding 23 and 6 would result in adding the 3 and 6 then adding the 2 and 6 for a total of 89. It must be noted, however, that it may be that these errors only appear when using standard written methods, not mental or alternative written methods. The examples presented by Ashlock use a standard written method.

There has been research on the *application* of addition with decimal fractions with a standard written method. In summarising research from the early 1980s, Hiebert and Wearne (1985, p.176) contended that, “by the third or fourth grade, many children do school mathematics by applying memorized rules to manipulate symbols”. The results of their research led them to conclude that written strategies for operations with decimal fractions are enacted by applying memorised rules by students in Grade 5 to Grade 9. They hypothesised that students learn a sequence of rules based on recognising or arranging particular configurations of numbers and symbols, and the arrangement itself triggers a remembered rule. An example rule they provided is “line up the decimal points” when performing addition. The rule demands a certain configuration for vertical addition, while the configuration reassures the student that the problem is now ready to be solved.

Hiebert and Wearne (1985) also suggested that there are three ways in which the rules are activated: one is recognising the type of operation required; a second is how well practised the rule is; and a third is how recently the rule was taught. Something that may interfere with activation is how visually similar a given problem is to another problem. Hiebert and Wearne contended that if the similarity is very close then the earlier and presumably better known rule will take precedence. For example, the rule for aligning the right-most digits for whole-number addition will be misapplied to a problem such as 1.23 + 45.6. For addition of decimal fractions where both addends involve hundredths, this particular rule should not cause difficulties, regardless of whether decimal fractions were first studied in a money context or as “bare number” problems.

Regarding mental computation and alternative written methods for adding decimal fractions, some studies point to the idea that students’ mental or alternative written methods with decimal fractions mimic those used for whole numbers, or make use of whole numbers in some way. Irwin and Britt (2004) found that some students were able to apply strategies used for whole numbers to decimal fractions, though with less accuracy than for whole numbers.

Caney and Watson (2003) found that some students split decimal fractions into whole numbers and the fractional parts and then worked on each type of number separately. Other students removed the decimal point to produce whole numbers, then added the whole numbers and reinserted the decimal point into the total. Given the range of misconceptions that students may have about decimal fractions, together with the range of ability that students have with whole number addition, further research on this topic seems warranted (Callingham & Watson, 2004, 2008).

Contextualising Problems

Context does not have one single meaning but can cover a range of possible interpretations ranging from the inclusion of a bare minimum of extra information drawn from real-life (such as measurement units) to the broader context in which learners work as they try to solve mathematics problems.

Proponents of the Dutch *Realistic Mathematics Education* program (RME) argue that mathematics should be grounded in reality first, rather than working with an abstraction which is then applied to a realistic problem. Problems can be real in the sense that they are meaningful and “imaginable for the students”, even if they involve fantasy settings such as fairies or, somewhat counter-intuitively, “bare-number” problems such as “1.30 + 1.20 = ?” (van den Heuvel-Panhuizen, 2005, pp. 2-3). Superficial use of contexts may only be an attempt to “dress up” bare-number problems instead of adding something to the actual problem. This use of context may happen with pictures or words, with a given context being able to be swapped for any other with no discernible effect on the desired answer (van den Heuvel-Panhuizen, 2005, p. 4). This idea was reiterated by Beswick (2011) who, in her review of research of context in mathematics education, made a distinction between “real-life” problems where a veneer of context is applied, and “authentic” or “situated” problems where problems are either more involved or resonate more with the person solving it. Wedege (1999) seemed to argue that context had to be considered as consisting of two main levels: “task context” is what the *problems* are in (i.e., a nominally realistic context), while “situation context” is what *learners* are in, with all the environmental, pedagogical, social, historical and other broader factors impacting on them.

Together with a variety of interpretations of what context involves, there are also a number of aims for including context. Some major themes that appear in the literature include:

- improving understanding of concepts (Beswick, 2011; Boaler, 1994; Putnam & Borko, 2000; Sullivan, 2011; van den Heuvel-Panhuizen, 2005)
- accuracy (Beswick, 2011)
- improving attitudes to mathematics generally (Beswick, 2011; Boaler, 1994)
- fostering mathematical thinking and reasoning motivating students by showing applications of mathematics they are learning (Beswick, 2011; Boaler, 1994; Sullivan, 2011)
- teaching about issues that an authority figure thinks are important demonstrating how mathematics is actually tackled in non-school contexts (Beswick, 2011)
- promoting transfer of “school mathematics” to vocational and everyday applications (Beswick, 2011; Boaler, 1994)

Despite the goals for using contextualised problems, there appear to be unresolved issues in actually using them. As Boaler (1993, p. 370) put it, “Contexts have the power to form a barrier or bridge to understanding”. A barrier can occur in various ways including the context being too far removed from the imaginable reality of the students (Boaler, 1993, 1994; Irwin, 2001; Wubbels, Korthagen & Broekman, 1997); the context triggering real-life knowledge that contradicts the assumptions of the problem writers (Boaler, 1993, 1994; Paterson & Bana, 2005; van den Heuvel-Panhuizen, 2005); or students recognising or assuming that the context is irrelevant given the larger environmental context of working in “textbook reality”, where the real goal is to get marks (Beswick, 2011; Boaler, 1994; van den Heuvel-Panhuizen, 2005; Wedege, 1999; Woodward, 2004).

Concerning the specific topic of money, at least one study has found that students performed better on a range of conceptual and procedural problems when the decimal fractions were contextualised as money (e.g. \$1.12 + \$3.39 + \$4) than when they were not (Rittle-Johnson & Koedinger, 2002). The types of benefits found by Rittle-Johnson and Koedinger may be harder to realise in an Australian context because an impediment to working with money problems in Australia is the absence of one-cent coins. Understanding that a cent is a hundredth of a dollar now relies on three abstract concepts: that of a dollar, that of a cent, and that of the relationship between a cent and a dollar. Prices involving any cent-amount (e.g. \$4.99) may still be advertised despite there being no physical coin to pay the amount in full. Consequently, prices paid for with cash are rounded to the nearest five cents. Sowder and Sowder (1998), in summarising the work of Terezinha, Carraher and colleagues, noted that placing operations in a money context improved accuracy and promoted more mental computation than when they were not placed in any context at all.

Even if students are successful with a problem in a given context, the transfer of any resulting knowledge or skills to new situations is not guaranteed. Although transfer has been an important topic in research for many years, there is little consensus on how to enable it to occur, possibly because of an assumption that it would naturally occur because the learning was contextualised (Anderson, Reder & Simon 2000; Prawat, 1989). There may have been some over-emphasis on the idea that knowledge and skills learnt within a school context cannot be transferred to outside-school contexts. By assuming contextualised learning is the “solution”, non-contextualised learning has been seen as the “problem” (Anderson, Reder & Simon, 2000).

So although context problems may have the potential to improve understanding and motivation, Beswick (2011, p.387) argued:

> …enthusiasm for context problems appears to be in advance of the evidence for their effectiveness… There is still much that is poorly understood about how contexts assist students to make sense of mathematics and which contexts are most effective in different circumstances.

Current Statutory Requirements

Context appears to have been a consideration in the writing of curriculum expectations in Queensland since at least 2004. At the time of the study, Queensland teachers were transitioning from a state syllabus to a national curriculum. Departmental advice (QSA, 2010a) was for schools to follow the existing state-based materials while staff became familiar with the new Australian Curriculum (ACARA, 2011). The existing materials comprised the Queensland Essential Learnings and Standards (ELS) (QSA, 2008) and the Queensland Year 1 to 10 Mathematics Syllabus (QSA, 2004). Both documents were expected to inform teachers' decisions about what to teach and when to teach it (QSA, 2010b). The version of the Australian Curriculum (AC) available during the period covered by the study has since undergone some slight revision.

Table 1 shows the different statements in each document relating to the representation and addition of money and of decimal fractions. Money problems sometimes involve calculating change which, although mathematically a subtraction problem, is often solved in practice and in classroom by counting on which is an additive process. The ELS go into considerable detail on different types of transaction and other money concepts. Table 1 only shows those statements most related to addition. Overall, the three documents stipulate or at least suggest that addition with money should occur earlier than the same operation with NCX problems involving hundredths. However, precise detail is not provided as to the types of money amounts that should be used, whether regrouping occurs or why operations with money should occur first.

Table 1
Curriculum References to Addition of Money and Decimal Fractions

Document	Year level	Statement
2004 Syll.	2-3	"When using money to purchase goods, they tender different combinations of notes and coins." (p. 19)
		"tendering cash for purchases" (p. 43)
		"reading and recording dollars and cents" (p. 43)
	4-5	"…solve addition and subtraction problems involving whole numbers and decimal fractions in context…" (p. 19)
		"[Identifying] equivalent values [with money]" (p. 43)
		"Conventions [with money]… [including] rounding totals for cash purchases" (p. 43)
		"change [i.e. money]" (p. 43)
		"[Addition of] decimals to 2 places in context with the same number of places" (p. 45)
		"[Subtraction involving] mental computations with money (change)" (p. 45)
	6-7	"…solve addition and subtraction problems involving whole numbers and common and decimal fractions…" (p. 20)
		"purchases…; budgets" (p. 44)
		"[Addition of] decimal fractions including different numbers of decimal places" (p. 46)

Table 1 (Continued)

2008 ELS	3	"Estimation of close values, e.g. using $5 note when the cost is $4.75" (p. 2)
	4	"spending plans, equivalent amounts" (p. 2)
		"simple financial records, e.g. list of expenditure with the leftover balances from savings, simple electronic spreadsheet" (p. 2)
	5	"addition and subtraction of… decimal fractions to hundredths" (p. 1)
		"names of coins and notes, change" (p. 2)
		"simple financial records" (p. 2)
	6	"addition and subtraction of… decimal fractions to hundredths" (p. 1)
		" budgets; financial records, e.g. table of savings, expenses and balances, electronic spreadsheet" (p. 2)
	7	"budgets; financial records" (p. 2)
2011 AC	2	"Count and order small collections of Australian coins and notes according to their value"
		"counting collections of coins or notes to make up a particular value, such as that shown on a price tag"
	3	" Represent money values in multiple ways and count the change required for simple transactions to the nearest five cents"
	4	"Solve problems involving purchases and the calculation of change to the nearest five cents with and without digital technologies"
	5	"Create simple financial plans"
	6	"Add and subtract decimals"
	7	[not applicable]

Note: 2004 Syll. = 2004 Syllabus (QSA, 2004); 2008 ELS = 2008 Essential Learnings and Standards (QSA, 2008); 2011 = 2011 Australian Curriculum: Mathematics (ACARA, 2011)
Note: The Australian Curriculum is an online document that varies according to user preferences: there is no printed copy. Thus, no page or paragraph numbers were available for citation.

Examples from Textbooks

Textbooks, also known as workbooks, are one way that curriculum is enacted. Data from the Trends in Mathematics and Science Study (TIMSS) conducted in 2007 revealed that 76 per cent of Year 4 teachers in Australia reported they use a textbook as either a primary or supplementary resource when teaching mathematics (Mullis, Martin & Foy, 2008). Although this figure is not as high as in some other countries (e.g., 92 per cent of US teachers reported the same), it is a substantial proportion, suggesting textbooks have an important influence on Australian primary teachers' practice.

Textbooks available in Queensland in 2011 mirrored the sequence laid out by the curriculum documents discussed earlier. Popular texts included:

- *GO Maths* (written for the 2004 syllabus)
  (Burnett & Irons, 2007a; Burnett & Irons, 2007b)

- *iMaths* (written for the 2008 Essential Learnings)
  (Linthorne, Smales, Lightbourne & Rheeder, 2010a; Linthorne, et al., 2010b)

- *Maths Plus* (written for the 2004 syllabus and 2008 Essential Learnings)
  (O'Brien & Purcell, 2009a; O'Brien & Purcell, 2009b)

- *New Signpost Maths* (written for the 2008 Essential Learnings)
  (McSeveny, Parker, Collard, McSeveny & McSeveny Foster, 2009a; McSeveny, et al., 2009b)

- *Targeting Maths* (written for the 2004 syllabus)
  (Turner, 2008; Turner, 2010)

Unsurprisingly, given the lack of fine detail in the Queensland-specific curriculum documents, these textbook series all have different approaches to operations with CX and NCX problems. An analysis of the Year 4 and Year 5 student books for each series revealed some considerable differences in sequence and format regarding decimal hundredths as shown in Table 2 and 3 below.

Table 2 shows that most textbooks for Year 4 followed the curriculum documents and kept addition to money contexts. Some of the questions were presented as pictures and others were presented as purely symbolic expressions. The ranges for totals varied greatly and the recommended time of year and frequency to teach addition in money formats were also diverse.

Table 2
Occurrence of CX and NCX Addition in Textbooks - Year 4

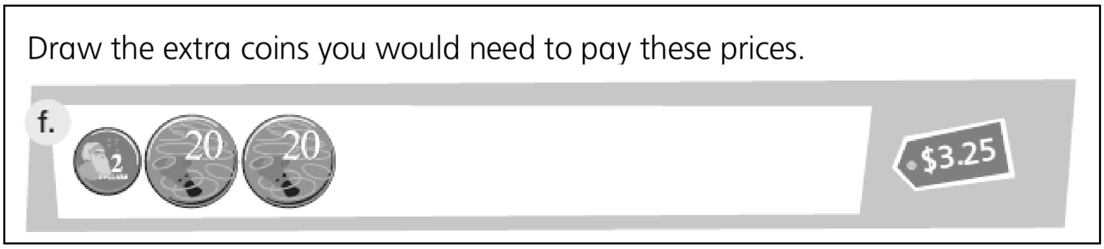
	Suggested sequence	Format	Maximum total
CX problems			
GO Maths	Term 3	Pictorial	$5.00
iMaths	Not stated	Symbolic	$9.00
Maths Plus	Terms 2, 3, 4	Pictorial and symbolic	$9.00
New Signpost Maths	Term 2	Symbolic	$99.00
Targeting Maths	Terms 1, 3, 4	Pictorial and symbolic	$20.00
NCX problems			
GO Maths	Does not occur	n/a	n/a
iMaths	Does not occur	n/a	n/a
Maths Plus	Does not occur	n/a	n/a
New Signpost Maths	Does not occur	n/a	n/a
Targeting Maths	Term 3	Symbolic	9.00

Table 3 shows that the Year 5 textbooks had similar differences to Year 4 regarding representation, pacing and ranges for totals. Most textbooks, however, were in accordance with the ELS (QSA, 2008) for the introduction of NCX decimal fractions.

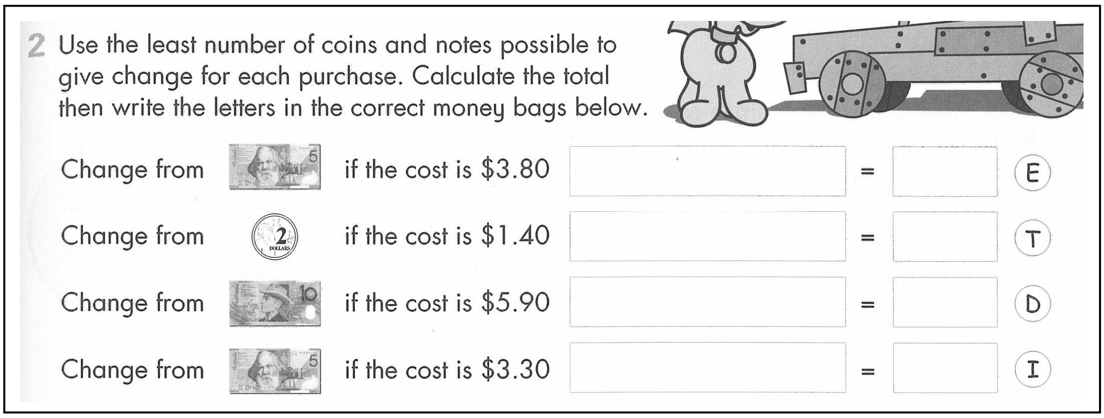
Table 3
Occurrence of CX and NCX Addition in Textbooks - Year 5

	Suggested sequence	Format	Maximum total
CX problems			
GO Maths	Term 3, 4	Pictorial and symbolic	$50.00
iMaths	Not stated	Symbolic	$99.00
Maths Plus	Term 3, 4	Symbolic	$19.00
New Signpost Maths	Terms 1, 2, 3	Symbolic	$99.00
Targeting Maths	Terms 1, 2	Pictorial and symbolic	$9.00
NCX problems			
GO Maths	Does not occur	n/a	n/a
iMaths	Not stated	Symbolic	800.00
Maths Plus	Term 4	Symbolic	99.00
New Signpost Maths	Term 1, 2, 3	Symbolic	99.00
Targeting Maths	Term 4	Symbolic	20.00

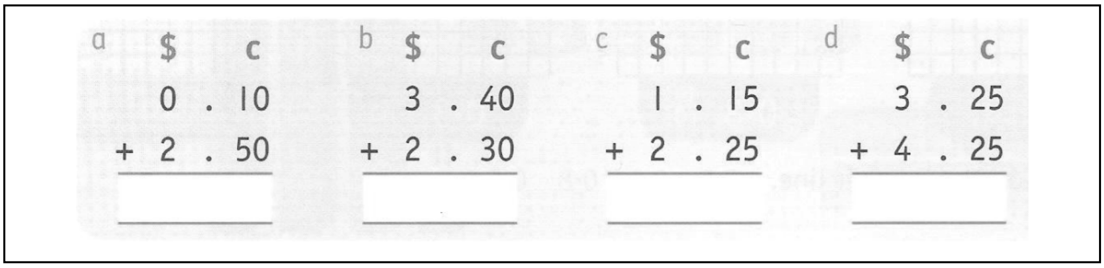
Figure 4 shows examples that represent the variety of questions from the textbooks listed in Tables 2 and 3. More examples can be found in Appendix A.



(a)



(b)



(c)

Figure 4. Example questions drawn from: (a) Burnett & Irons (2007b, p. 119); (b) Linthorne et al. (2010a, p. 54); and (c) Turner (2010, p.22).

Summary

In this chapter it was noted that some research suggests that the use of money, with its strong distinction between dollars and cents as types of units, may impact negatively on how students conceptualise decimal fractions generally. It was also observed that there appears to be a dearth of large-scale research into the methods used to add decimal fractions involving hundredths and the rate of accuracy for such operations. Placing problems in contexts appears to be desirable for a range of reasons but runs the risk of creating unintended obstacles to learning. Queensland curriculum documents and the recently released Australian curriculum support the teaching of operations with money before generalised decimal fractions. There is little guidance, however, on if or how the relationship between contextualised and non-contextualised decimal fractions should be explained. Textbooks consequently show a variety of representations of NCX and CX problems.

In Chapter 3 the methodology for the study is described, detailing the identification of potential participants, construction of the test papers, testing procedures and creation of the marking guides.

CHAPTER 3: METHODOLOGY

In the previous chapters it was suggested that the use of money to represent decimal fractions involving hundredths is poorly researched and that potential positive effects gained from contextualising problems may have unintended negative impacts. The two questions that guided further investigation into these topics are:

1. How does accuracy with addition differ when decimal fractions to hundredths are written with dollar signs compared to when they are not?
2. How do addition methods differ when decimal fraction problems using hundredths are contextualised with dollar signs compared to when they are not?

This chapter describes how Queensland students in Years 4 and 5 were selected for the study, how the questions were chosen for the test papers (which were novel test instruments) and what steps were taken to reduce the effect of extraneous variables. The discussion of the marking guides also highlights unexpected effects of contextualising decimal fractions as money.

Research Design

A cross-sectional design was chosen to provide a snapshot of Year 4 and Year 5 students' ability to compute decimal addition problems when the problems were presented with and without context. The design provided a random assignment of participants to two groups: context (CX) and non-context (NCX).

Participants

Participants in this study were 86 Year 4 and 75 Year 5 students in state schools in or near Brisbane, Queensland. The choice of those year levels was a reflection of when transactions with money were promoted in Queensland curriculum documents and textbooks (see Chapter

2 for details). Government schools only were chosen because the state system covered more than 70 per cent of schools in Queensland in 2010 (Australian Bureau of Statistics (ABS), 2012).

Schools were invited to participate based on the following requirements:

- They had mean results of between 312 and 445 on the Year 3 Numeracy section of the 2010 NAPLAN tests.
- They were sufficiently large enough to have single classrooms of Year 4 students and also of Year 5 students (i.e., not multi-age/composite classes).
- They were within a 60-minute drive from the researcher's home or work address (i.e., a convenience sample).

The Queensland state mean for the 2010 Year 3 Numeracy section of the NAPLAN test was 378.5, with a standard deviation of 65.9 (ACARA, 2010a), thus the schools chosen were within one standard deviation of the mean.

The Index of Community Socio-Educational Advantage (ICSEA) is a measurement tool developed by ACARA to identify schools by the socio-economic status of their students (ACARA, 2010c). Although the ICSEA scores of schools could have been used to ensure representativeness along socio-economic lines, the ICSEA scores and NAPLAN scores have a moderately strong, positive correlation (ACARA, 2010c).

Based on the characteristics described above, 78 schools and colleges were initially invited to participate by consulting their NAPLAN 2010 scores (QSA, 2010b). Of those approached, eight schools responded positively with 13 classes involved. The range of the 2010 NAPLAN Year 3 Numeracy means for the schools was 368 to 420 (M = 388.9, SD = 16.8). Hence the mean NAPLAN score of the schools was slightly above the Queensland mean. All classes had at least 20 students but, within the 13 classes that participated, the number of students actually involved in the study ranged from 5 to 24. In other words, in some classes almost all students participated and in others only a small number did.

Participating students in each class at each school were split randomly into two approximately equal groups. Group A (n = 80) received the NCX test, Group B (n = 81) received the CX test. A block-randomisation procedure was used to allocate participants to the groups (Gould, 2002). Because whole classes were being used, students were already blocked by a common attribute. The procedure was as follows:

- The class names were listed in alphabetical order by first name.
- On the day of testing, the list was checked to see which participants were present and the list was adjusted as needed.
- The first participant present on the list was assigned to Group A, the second to Group B, the third to Group A and so on.
- Once a classroom participated in the study, the next classroom involved had participants assigned in the same way, continuing the allocation from the previous classroom based on which group had the most members. For example, if the final numbers of participants in Classroom 1 were 13 in Group A and 12 in Group B, then the allocation process for Classroom 2 would begin with the group that had the lesser total, i.e., Group B.

Instruments

As noted in Chapter 2, no large-scale research could be found that focussed specifically on testing with decimal fractions involving hundredths. For this reason tests were designed specifically for the study. In order to keep the test length reasonable regarding the amount of class time it took and so as not to overwhelm the students, the test length was restricted to no more than 40 minutes. Two tests were created, with each test having 30 questions: five whole number questions and 25 decimal fraction questions. This number of questions and test duration are consistent with other commonly used tests such as NAPLAN (ACARA, 2010b).

The CX test (Appendix B) placed a dollar sign ($) in front of each addend in the decimal fraction part of the test. The NCX test (Appendix C) did not have any extra symbols and neither tests had dollar signs in the whole number questions. NCX and CX tests were identical apart from the NCX test having decimal fractions without contextual symbols and the CX test having the same decimal fractions written in a dollar-and-cent format. For example, the NCX test had "1.30 + 1.20 = _____" so the CX test had
"$1.30 + $1.20 = _____".

To choose a suitable collection of decimal fraction questions, the full range of expressions that are possible when combining two addends comprising of ones, tenths and hundredths were identified (ones are also called units). Only ones, tenths and hundredths were chosen for the decimal place values and all numbers were less than 10 to avoid overload by having larger numbers. The different ways that regrouping could occur with these expressions were also determined. Of the 51 general expressions that were applicable, 25 were chosen for the tests. Appendix D describes the selection process for these 25 questions in full, but in brief the decimal fraction questions were selected to best represent the range of:

- regrouping situations, covering regrouping of hundredths, tenths or both;
- expressions that seemed realistic, slightly unrealistic or very unrealistic in everyday money contexts;
- expressions that would reveal whether students strategically adjusted some numbers before adding them (such as changing 0.99 + 0.70 to 1.00 + 0.69);
- expressions where students might be tempted to round amounts to the nearest multiple of 5, as happens in Australian cash transactions.

Of the decimal fraction questions, expressions were judged as realistic, unrealistic or very unrealistic in a money context (i.e. purchases). The underlying question guiding this judgement was “Would it be likely to find two items with these prices in a supermarket?” The following criteria were used to help determine the answer to the question:

- Realistic questions had both addends as whole-and-fraction amounts (e.g., 1.34 + 1.25).
- Unrealistic questions had one addend as a whole-and-fraction amount and the other had a zero in the ones place (e.g., 1.34 + 0.25), or both had a zero in the ones place (e.g., 0.34 + 0.25).
- Very unrealistic questions had one or both addends as a zero in the ones and tenths place (e.g., 1.34 + 0.05).

The first three decimal fraction questions on the test were chosen as relatively easy problems as they did not require regrouping, they used multiples of five and were realistic as defined above. Thereafter, questions were chosen to cycle between unrealistic, very unrealistic and realistic to ensure there was an even mix of question types throughout the tests. For consistency, all questions on the tests were written with the greater addend first. For example, “2.75 + 1.20” was used instead of “1.20 + 2.75”.

The whole number subset of five questions was selected to examine these aspects:

- Question A (40 + 30): how addition involving zeros was attempted.
- Question B (65 + 24): whether multi-digit addition without regrouping could be done.
- Question C (74 + 58): whether multi-digit addition with regrouping could be done.
- Question D (67 + 5): how uneven (ragged) addition was attempted.
- Question E (198 + 80): whether students strategically adjusted some numbers before adding them.

Although success or failure with these five questions could directly relate to performance on particular decimal fraction test items, a comparison of the two sets of questions was not included in this study. Because many of the decimal fraction questions had features reflected in the whole number questions, achievement on the whole number questions could provide some baseline comparisons between the NCX and CX groups.

The CX and NCX tests were novel measurement instruments and were checked for validity and reliability (Marczyk, DeMatteo & Festinger, 2005). Marczyk et al. (2005, p. 164) defined validity as "whether the approach to measurement used in the study actually measures what it is supposed to measure". To examine the different types of validity that were applicable, Babbie's (2007) definitions were consulted. The tests measured accuracy in a written format only and the types of written methods that were used and as such had face validity in terms of the intention of the study. The tests addressed a narrow range of content (i.e. only addition of decimal fractions of a certain type) to provide construct validity. Lastly, because the tests used a wide range of questions about the content they had content validity because they adequately covered the range of possible content.

Reliability is defined by Marczyk et al. (2005, p. 103) as "the consistency or dependability of a measurement technique". Following the guidelines by Marczyk et al. (2005) for minimising the impact of measurement error, all tests:

- had the same questions (aside from dollar symbols as appropriate) in the same order and format;
- had the same instructions;
- were distributed and completed under approximately the same conditions.

Further, the completed tests were assessed by one coder (the researcher), with a second coder used for a sample of tests to ensure interpretation was accurate. Data entry was also audited. These processes aimed to improve the reliability of the tests.

Procedure

Potential participants and their parents received information and consent forms before the test sessions. Students who completed the consent forms as well as their parents became participants. Once participants were allocated to groups as described previously, the two tests were handed out by the researcher. After all the tests were distributed a script (Appendix E) with instructions for completing the tests was read aloud. The students were asked to answer questions using written or mental methods, but were not allowed to use calculators or counting on rulers. They were asked to show their thinking or working out. The tests were completed by participants in their ordinary classrooms, apart from two occasions when participants completed the tests in a separate room. All rooms were set up for test conditions, where desks were separated and students worked silently and independently. Rearranging of desks was expected to be slightly disruptive, so teachers were asked to rearrange the desks before testing, but when this did not occur the students rearranged the desks just before testing.

Once a participant completed the test the researcher reviewed the answers. If a question was incomplete or not attempted, or if the answer was not clear, the researcher drew the participant's attention to it and gave the student another opportunity to complete the question. If a participant declined the researcher did not ask again. Once a test paper was complete the researcher collected it. In most classes, all students in a class did the tests although, as mentioned above, sometimes only a small fraction of the class participated in the study. The decision to have non-participants do the tests was left to the classroom teacher to make and, when this occurred, the cover sheets of the tests of non-participating students were removed after completion and the tests were left with the teacher. That is, the test answers of students who had not agreed to inclusion in the study were not included.

After all test papers for a classroom were completed, a database was created to record the classroom, year level, school, sex, age (in years only) and group allocation of participants. Each participant, classroom and school was assigned an ID number with the participant ID recorded on the corresponding tests. Results were entered into the database after marking and aggregated, de-identified data was provided to the classroom teacher (i.e. only results for students for participating students as a whole group were given). The cover sheets were then removed and the consent forms kept in a secure location.

A trial study involving a Year 4 class (n = 24) and Year 5 class (n = 17) confirmed that all procedures and instruments worked as planned. As such, the results from the trial study were pooled with subsequent participants for analysis.

Extraneous Variables and Controls

Because of the randomisation process described earlier, participants in any given classroom were divided into the NCX and CX groups. Any extraneous variables that affected a particular classroom (for example, teacher experience or environmental conditions) hence affected the performance of participants in both groups in that classroom. This process acted to improve control (Gould, 2002). Further controls for the extraneous variables listed below were taken and were based on recommendations by Gould (2002) and Marczyk et al. (2005).

Physical environment. Classrooms were organised for test conditions (i.e. separated desks, no talking or passing notes). Although this may have been disruptive to participants' usual routines the effects were distributed across both groups.

Test administration. The tests were completed in a single sitting of 40 minutes duration. The time of day was restricted to early morning to noon as after lunch sessions are, from experience, more difficult times for focussing students' attention. Similarly, no testing sessions were conducted on a Friday.

Experimenter features. The tests were administered by only one person (the researcher) using a script (Appendix E) to explain procedures. This aimed to reduce any variation in instructions.

Data Analysis

To address the research questions, the data was analysed for accuracy and type of method. Aside from coding for demographic information, coding of responses was necessary for both sets of questions and is described below. Analysis of age or gender effects was conducted, though minimally and only included as additional data.

Students showed a greater than anticipated range of responses that had not been identified in the pilot testing process. These were not simply diverse methods of completing a question but different answers. Some students showed the correct answer in the answer space next to the original expression (i.e. the reported answer) but showed another answer in their written calculation (i.e. the method answer). Others did the reverse. Some participants using the CX test reported an answer in cents only, or as dollars and cents, or without the use of the dollar symbol and all these responses had to be determined as being either correct or incorrect. Consequently, a marking guide (Appendix F) was developed as the test papers were reviewed. As new responses were identified, changes were made to refine the marking guide and previously recorded results for participants were re-examined and adjusted if necessary. These were classified as Response Types. In the marking guide, the "reported answer" is the total that the students wrote beside the existing printed expression. The "method answer" is the total that students recorded after they used a written method. These are shown in Figure 5.

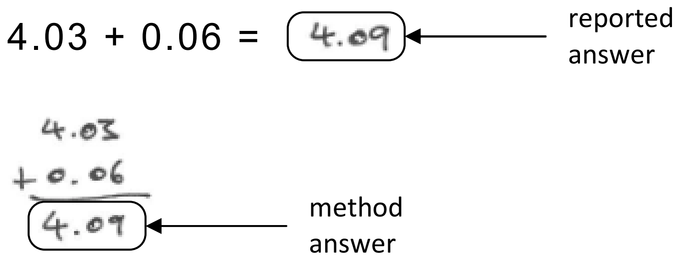


Figure 5. Examples of reported answer and method answer.

Arising from the creation of the marking guide was a dilemma regarding whether answers that had incorrect money notation should be judged as correct or incorrect. These responses did not clearly fall into the classes of major error or minor errors as defined in Chapter 1. As the notation discrepancies could be caused by a conceptual error or simply as an oversight, it was decided that the data analysis would involve two scenarios: one where those responses were marked as incorrect (Scenario 1) and another where they were marked as correct (Scenario 2).

Identifying a response as using a standard written method or an alternative written method was more straightforward than determining accuracy. Only four codes were used to record a response. These were classified as Method Types as follows:

- not done;
- no method shown;
- standard written method (as defined in Chapter 1);
- alternative written method.

Alternative written methods included those where sentences were written with words and also those where a non-standard vertical format was used for all or part of the calculation.

Examples of these methods are shown in Chapter 1 and in the marking guide in Appendix G. No further analysis of individual alternative written methods was conducted, only the approach of using such methods generally, as the scope of such an analysis was beyond the practical limits of this study.

Reliability of Marking Guides

The reliability of the marking guides for CX and NCX tests was established by randomly selecting sixteen test papers from each participant group (i.e. about 20 per cent of all test papers) for the researcher and a second coder to analyse for types of answers and type of method. Afterwards, any differences in coding were discussed and codes were adjusted if necessary until the two coders agreed completely on how each test paper should be coded. Disregarding instances where codes were further refined in meaning, coding errors made by the first coder were then analysed to give an estimate of how accurate the complete coding of all 161 test papers would be.

Of the 960 Response Types, eight (about 0.8 per cent) were coded incorrectly by the researcher. However, none of the coding errors resulted in answers being marked correct when they were actually incorrect or vice versa.

Of the 960 Method Types, 15 (about 1.6 per cent) were coded incorrectly by the researcher. All but one of the errors involved coding an alternative written method as a standard written method. The remaining error involved coding a standard written method as no method.

Both sets of errors (Response Type and Method Type) seemed to be transcription errors by the researcher rather than a misinterpretation of the codes. Sixteen test papers from each participant group were also randomly selected to determine the error rate for transcribing the codes into a database. No errors were found for either group.

Ethics

In accordance with requirements, an ethics application was made to and approved by the University of Tasmania's Human Research Ethics Committee for this study (reference H11797). Additional ethics approval was also sought from and given by the Queensland Department of Education and Training (reference 550/27/1101). All relevant guidelines and requirements from both bodies were followed during the study. In accordance with Queensland Department of Education and Training requirements, consent was sought from students and parents to enable student participation. Relevant documents are provided in Appendix H.

Summary

This chapter described the study methodology and the processes for how to test whether accuracy and method choice is affected by context. Various measures were taken to control extraneous variables to ensure the two test groups were equivalent. The creation of the marking guides for the tests necessitated considering two scenarios for interpreting accuracy. The fact that two scenarios were necessary points to the idea that the inclusion of money notation at a simple level affects assessment of student responses. In Chapter 4 the results from the study are given. In line with the research questions, data on accuracy and method choice are given.

CHAPTER 4: RESULTS

Previous chapters outlined why modelling decimal fractions in a money context may have unintended impacts on how decimal fractions are thought of generally. A randomised two-group cross-sectional design was chosen to examine the accuracy and method choice for decimal fraction addition problems placed in a money context or not. The creation of the marking guide for determining accuracy resulted in establishing two scenarios for data analysis. This chapter gives the results of the study.

Demographics

Table 4 shows the demographic make-up of the students in each group after the allocation process. As can be seen, the number of students in each group for each year level is almost the same although the split by gender is uneven.

Table 4
Year Level and Sex of Participants, by Condition

		NCX			CX		
Sex	*n*	Year 4	Year 5	Total	Year 4	Year 5	Total
Male	76	16	21	37	21	18	39
Female	85	27	16	43	22	20	42
Total	161	43	37	80	43	38	81

Accuracy

Whole numbers. First the whole number components of the tests were considered to see whether the groups were equivalent in their ability to do addition problems presented in a no context format. Table 5 shows that the students assigned to the CX group appeared to perform slightly better with the whole number problems although effect sizes were small and below the level that Hattie (2007) would regard as important in education. As the whole number questions were identical on both types of tests, it is likely the difference between groups occurred by chance during the allocation process. Overall, the groups could be considered equivalent at computation.

Table 5
Effect of Context - Whole Numbers

	NCX			CX			
Year level	*n*	*M*	*SD*	*n*	*M*	*SD*	*d*
Year 4	43	4.26	1.03	43	4.51	0.83	-0.27
Year 5	37	4.49	0.90	38	4.68	0.57	-0.26
Both years	80	4.36	0.97	81	4.59	0.72	-0.27

Note: The highest possible score was 5.

Decimal fractions. Accuracy with the decimal fraction questions of the tests was examined next. As described in Chapter 3, two scenarios were taken into account when marking this section. Scenario 1 assumed that answers such as “50”, “0.50c” and “$0.50c” were incorrect responses to the question “$0.25 + $0.25 = ___”. Scenario 2 assumed that such answers were correct, with some notation errors. Response Type 240 was the code for such answers. Table 6 shows that the NCX group achieved greater accuracy in both scenarios, with small to moderate effect sizes in Scenario 1 but almost no effect size in Scenario 2. The results from the whole number section suggested that the CX group would perform slightly better than the NCX group so this finding is unusual. This finding suggests that the students use different ways of thinking about whole-number calculations compared to calculations involving decimal fractions. It also indicated that the issue was less to do with computational skills because the differences were almost negligible when the notation errors were discounted in Scenario 2.

Table 6
Effect of Context - Decimal Fractions

	NCX			CX			
Year level	*n*	*M*	*SD*	*n*	*M*	*SD*	*d*
Year 4							
Scenario 1	43	21.51	3.88	43	19.84	6.85	0.30
Scenario 2	43	21.51	3.88	43	21.12	5.24	0.09
Year 5							
Scenario 1	37	22.95	3.67	38	21.39	4.48	0.38
Scenario 2	37	22.95	3.67	38	22.47	3.01	0.14
Both years							
Scenario 1	80	22.18	3.83	81	20.57	5.88	0.32
Scenario 2	80	22.18	3.83	81	21.75	4.37	0.10

Note: The highest possible score was 25.

Response Types

In addition to the data about the accuracy of participants' answers, the ways in which answers were presented yielded important information about the influence of context. For quick identification, the Response Types were coded with a "0" at the start if they were incorrect, a "1" if they were correct, and a "2" if they were not clearly correct or incorrect. Appendix F elaborates on each Response Type and provides examples, but the Response Types are summarised here in Table 7.

Table 7
Descriptions of Response Types

Response Type	Description
ND	the question was not attempted at all
110	completely correct answer with no errors
010, 011, 012 and 013	various errors relating to such things as inaccuracy with addition of one-digit numbers, using a written method incorrectly, or omitting decimal points
120	an answer that is conceptually correct but which may be missing a "0" in cases such as "2.50" or "0.15"
020	various errors relating to place value and regrouping
130	an answer that is rounded after addition
030	an answer that is rounded before addition
140, 141and 240	answers that involve various errors in money notation

Whole numbers. Table 8 shows the NCX and CX groups used approximately the same number of Response Types as each other for the whole number questions. So, in the types of responses that each group made, the groups were roughly equivalent in this measure. In both groups the most common error was 011 which specifically addressed adding two one-digit numbers as part of the larger process of adding multi-digit numbers. For example, when adding 65 and 24, a student may have added the 5 and 4 together for an answer of 7 instead of 9. The other Response Type that was relatively high in both groups was 012 which dealt with mechanical errors with using a written method. For example, if using a standard written method to add 67 and 5, the student may have aligned the 5 under the 6 or added 76 and 5.

Also of interest is that for both groups around 20 per cent of students in each group answered Question E (198 + 80 = ___) incorrectly. The Response Type groupings indicated this was mainly due to students making relatively simple errors, such as: miscalculating a sub-addition such as adding 8 and 0, or 9 and 8 (Response Type 011); making a procedural error such as forgetting a carry digit in a standard written method (Response Type 012); or not recording a method or using an unintelligible method (Response Type 010). Response Types 130, 030,140, 141 and 240 were not applicable to whole number questions so are not shown in Table 8.

Table 8
Frequency of Response Types - Whole Numbers

RT	Question a	b	c	d	e	Total	*M*
Year 4 NCX							
ND	0	0	0	0	0	0	0.00
110	42	39	30	38	34	183	36.60
010	0	0	3	0	1	4	0.80
011	0	2	6	3	4	15	3.00
012	0	2	0	2	3	7	1.40
013	0	0	0	0	0	0	0.00
120	0	0	0	0	0	0	0.00
020	1	0	4	0	1	6	1.20
Total	43	43	43	43	43	215	43.00
Year 4 CX							
ND	0	1	0	0	0	1	0.20
110	43	41	38	40	32	194	38.80
010	0	1	2	1	2	6	1.20
011	0	0	3	1	5	9	1.80
012	0	0	0	1	2	3	0.60
013	0	0	0	0	0	0	0.00
120	0	0	0	0	0	0	0.00
020	0	0	0	0	2	2	0.40
Total	43	43	43	43	43	215	43.00

Table 8 (Continued)

RT	Question a	b	c	d	e	Total	*M*
Year 5 NCX							
ND	0	0	0	0	0	0	0.00
110	35	33	33	34	31	166	33.20
010	2	2	2	1	3	10	2.00
011	2	2	2	1	3	10	2.00
012	0	1	1	2	1	5	1.00
013	0	0	0	0	0	0	0.00
120	0	0	0	0	0	0	0.00
020	0	0	0	0	0	0	0.00
Total	39	38	38	38	38	191	38.20
Year 5 CX							
ND	0	0	0	0	0	0	0.00
110	37	38	36	34	33	178	35.60
010	0	0	2	3	2	7	1.40
011	1	0	0	1	3	5	1.00
012	0	0	0	0	0	0	0.00
013	0	0	0	0	0	0	0.00
120	0	0	0	0	0	0	0.00
020	0	0	0	0	0	0	0.00
Total	38	38	38	38	38	190	38.00

Table 8 (Continued)

RT	Question a	b	c	d	e	Total	*M*
NCX							
ND	0	0	0	0	0	0	0.00
110	77	72	63	72	65	349	69.80
010	2	2	5	1	4	14	2.80
011	2	4	8	4	7	25	5.00
012	0	3	1	4	4	12	2.40
013	0	0	0	0	0	0	0.00
120	0	0	0	0	0	0	0.00
020	1	0	4	0	1	6	1.20
Total	82	81	81	81	81	406	81.20
CX							
IC	0	1	0	0	0	1	0.20
110	80	79	74	74	65	372	74.40
010	0	1	4	4	4	13	2.60
011	1	0	3	2	8	14	2.80
012	0	0	0	1	2	3	0.60
013	0	0	0	0	0	0	0.00
120	0	0	0	0	0	0	0.00
020	0	0	0	0	2	2	0.40
Total	81	81	81	81	81	405	81.00

Note: Totals of Response Types may be higher than the number of participants in each group as some participants made multiple errors.
Note: RT = Response Type; see Table 7 for explanations for other codes.

Decimal fractions. Table 9 shows the Response Types for the decimal fraction questions. Although there were some differences between the two groups in most Response Types, the most pronounced differences were with Response Types 140, 141 and 240, which involve money notation, as might have been expected. About 17 per cent of all Response Types for the CX group involved these three types. Consideration of the accuracy of Response Type 240 resulted in the creation of the two scenarios. Particularly noticeable for the CX group was the prevalence of Response Type 140 for Questions 8, 14, 21 and 23. These questions are shown in Table 10.

Table 10
Questions with High Frequencies of Response Type 140

Question	Problem
8	\$0.08 + \$0.07 = ___
14	\$0.18 + \$0.02 = ___
21	\$0.25 + \$0.25 = ___
23	\$0.76 + \$0.24 = ___

These four questions had the highest frequency of any Response Type (other than 110, fully correct). Response Type 140, which involved reporting amounts less than a dollar as cents only (e.g., \$0.08 + \$0.07 = 15c), was by far the most common Response Type. Roughly one-quarter of all CX participants displayed this response for each of the four questions.

For the NCX and the CX groups, Questions 6, 7, 8, 14, 15, 16 and 25 were among the questions that had the highest frequency of errors, major and minor. These questions are shown in Table 11, in NCX form. They all involve some regrouping.

Table 11
Questions with High Frequencies of Errors

Question	Problem
6	4.85 + 1.55 = ___
7	0.99 + 0.70 = ___
8	0.08 + 0.07 = ___
14	0.18 + 0.02 = ___
15	3.98 + 0.40 = ___
16	1.64 + 0.87 = ___
25	3.50 + 0.68 = ___

Lastly, the CX group had a greater number of questions left incomplete or not done at all, increasing in number towards the end of the test. The total number of students not completing a question was still relatively low, however. For example, only 5 out of 81 students (6%) in the CX group did not complete Questions 24 and 25.

Table 9

Frequency of Response Types - Decimal Fractions

	Question																										
RT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Total	*M*
Year 4 NCX																											
ND	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	3	0.12
110	41	39	34	41	37	31	35	34	37	40	39	40	38	36	37	28	38	38	36	36	37	39	38	35	34	918	36.72
010	0	0	1	0	1	1	0	0	1	0	0	0	0	1	1	2	0	1	1	0	1	1	0	1	1	14	0.56
011	0	1	2	1	1	1	2	4	2	1	1	1	0	0	0	8	1	1	2	2	0	1	1	1	4	38	1.52
012	0	0	1	0	1	6	0	0	1	0	1	0	2	1	3	4	0	0	1	1	0	1	1	3	1	28	1.12
013	1	2	3	0	1	2	2	3	1	2	2	2	1	2	1	1	2	2	1	2	3	1	1	1	1	40	1.60
120	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	3	0.12
020	1	1	2	1	3	2	3	0	2	0	0	0	2	1	1	1	0	2	2	2	1	0	1	1	1	30	1.20
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
030	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.04
140	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	4	0.16
141	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
Total	43	43	43	43	44	43	43	43	44	43	43	43	43	43	43	44	43	44	43	43	43	43	43	43	43	1079	43.16

Table 9 (Continued)

	Question																										
RT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Total	*M*
Year 4																											
CX																											
ND	1	1	1	1	1	0	0	0	0	1	1	2	1	1	2	3	4	4	4	4	4	4	4	5	5	54	2.16
110	34	33	33	33	36	26	31	27	34	34	32	32	31	26	24	25	23	32	31	32	22	32	31	28	25	747	29.88
010	2	0	0	0	2	7	8	2	1	1	3	2	4	1	4	5	1	2	3	1	2	1	2	3	4	61	2.44
011	1	2	1	1	0	4	1	0	1	0	0	0	1	0	5	3	0	1	1	1	0	0	0	0	2	25	1.00
012	0	0	2	0	1	3	0	0	0	0	1	0	1	0	1	4	2	0	0	2	0	0	0	0	1	18	0.72
013	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0.08
120	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	4	0.16
020	0	0	0	0	2	0	0	1	0	0	0	0	0	0	2	1	0	2	1	0	0	0	2	1	0	12	0.48
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
030	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
140	0	0	0	0	0	0	0	10	1	1	1	1	1	10	1	1	9	0	0	0	10	1	1	2	0	50	2.00
141	1	3	2	4	1	2	3	2	2	2	2	2	2	2	2	3	2	2	2	1	2	2	3	4	4	57	2.28
240	4	4	4	4	2	3	2	1	4	4	3	4	3	2	2	1	2	2	1	2	2	3	2	2	3	66	2.64
Total	43	43	43	44	45	45	45	44	43	43	43	43	44	43	43	46	44	45	43	43	43	43	45	46	44	1096	43.84

Table 9 (Continued)

RT	Question																									Total	*M*
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25		
Year 5																											
NCX																											
ND	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
110	33	35	35	35	35	27	33	33	32	34	36	33	32	34	33	31	35	36	33	33	35	36	35	33	30	837	33.48
010	2	0	0	1	1	6	1	1	1	1	1	1	1	1	2	3	1	1	1	4	2	0	1	0	1	34	1.36
011	2	0	0	1	1	6	1	1	1	1	1	1	1	1	2	3	1	1	1	4	2	0	1	0	1	34	1.36
012	0	0	2	1	0	2	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	1	1	1	4	16	0.64
013	0	1	0	0	0	0	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	8	0.32
120	1	1	0	0	0	1	0	1	0	2	0	0	2	1	0	0	1	0	2	0	0	0	0	0	0	12	0.48
020	0	0	0	0	1	0	1	0	1	0	0	1	0	0	1	0	0	0	1	0	0	0	0	1	0	7	0.28
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
030	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
140	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
141	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
Total	38	37	37	38	38	42	37	37	36	39	38	38	38	38	38	37	38	38	38	41	39	37	38	36	37	948	37.92

Table 9 (Continued)

	Question																										
RT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Total	*M*
Year 5																											
CX																											
ND	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
110	34	35	32	27	27	27	28	18	27	32	29	32	29	17	24	23	18	30	28	26	17	26	27	22	29	664	26.56
010	1	0	1	3	1	2	2	3	1	0	0	0	0	0	4	1	0	2	3	2	1	2	1	2	3	35	1.40
011	0	0	2	1	0	3	3	0	2	0	0	0	2	0	3	3	0	0	0	1	0	1	0	0	0	21	0.84
012	0	0	1	0	0	0	1	1	0	0	0	0	0	0	1	3	0	0	0	2	0	0	1	2	0	12	0.48
013	0	1	0	0	0	0	0	0	0	0	1	0	1	0	2	3	0	0	1	1	0	0	2	1	1	14	0.56
120	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.04
020	0	0	0	0	4	0	0	2	1	0	0	0	1	0	2	1	0	1	2	0	0	0	1	2	0	17	0.68
130	0	0	0	2	2	0	2	0	2	0	2	0	0	0	2	2	0	2	0	2	0	2	1	0	1	22	0.88
030	0	0	0	1	1	0	1	1	1	0	1	0	0	0	0	1	0	0	0	0	0	1	1	0	0	9	0.36
140	0	0	0	0	0	0	0	7	0	0	0	0	0	13	0	0	12	0	0	0	13	0	0	4	0	49	1.96
141	2	1	1	3	4	4	4	3	4	4	3	4	3	3	4	4	2	4	6	3	2	4	6	6	4	88	3.52
240	1	1	1	1	1	1	1	5	2	2	2	2	2	5	0	0	6	1	2	1	5	2	1	2	2	49	1.96
Total	38	38	38	38	40	38	42	40	40	38	38	38	38	38	42	41	38	40	42	38	38	38	41	41	40	981	39.24

Table 9 (Continued)

	Question																										
RT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Total	*M*
NCX																											
ND	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	3	0.12
110	74	74	69	76	72	58	68	67	69	74	75	73	70	70	70	59	73	74	69	69	72	75	73	68	64	1755	70.20
010	2	0	1	1	2	7	1	1	2	1	1	1	1	2	3	5	1	2	2	4	3	1	1	1	2	48	1.92
011	2	1	2	2	2	7	3	5	3	2	2	2	1	1	2	11	2	2	3	6	2	1	2	1	5	72	2.88
012	0	0	3	1	1	8	0	0	2	1	1	1	3	1	3	4	0	0	1	1	0	2	2	4	5	44	1.76
013	1	3	3	0	1	2	3	4	1	2	2	3	2	3	1	1	2	2	1	2	3	1	1	2	2	48	1.92
120	1	1	0	0	0	1	0	2	0	2	0	0	2	2	0	0	2	0	2	0	0	0	0	0	0	15	0.60
020	1	1	2	1	4	2	4	0	3	0	0	1	2	1	2	1	0	2	3	2	1	0	1	2	1	37	1.48
130	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
030	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.04
140	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	4	0.16
141	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.00
Total	81	80	80	81	82	85	80	80	80	82	81	81	81	81	81	81	81	82	81	84	82	80	81	79	80	2027	81.08

Table 9 (Continued)

	Question																										
RT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Total	*M*
CX																											
ND	1	1	1	1	1	0	0	0	0	1	1	2	1	1	2	3	4	4	4	4	4	4	4	5	5	54	2.16
110	68	68	65	60	63	53	59	45	61	66	61	64	60	43	48	48	41	62	59	58	39	58	58	50	54	1411	56.44
010	3	0	1	3	3	9	10	5	2	1	3	2	4	1	8	6	1	4	6	3	3	3	3	5	7	96	3.84
011	1	2	3	2	0	7	4	0	3	0	0	0	3	0	8	6	0	1	1	2	0	1	0	0	2	46	1.84
012	0	0	3	0	1	3	1	1	0	0	1	0	1	0	2	7	2	0	0	4	0	0	1	2	1	30	1.20
013	0	1	0	1	0	0	0	0	0	0	1	0	1	0	2	3	0	0	1	1	0	0	2	2	1	16	0.64
120	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	5	0.20
020	0	0	0	0	6	0	0	3	1	0	0	0	1	0	4	2	0	3	3	0	0	0	3	3	0	29	1.16
130	0	0	0	2	2	0	2	0	2	0	2	0	0	0	2	2	0	2	0	2	0	2	1	0	1	22	0.88
030	0	0	0	1	1	0	1	1	1	0	1	0	0	0	0	1	0	0	0	0	0	1	1	0	0	9	0.36
140	0	0	0	0	0	0	0	17	1	1	1	1	1	23	1	1	21	0	0	0	23	1	1	6	0	99	3.96
141	3	4	3	7	5	6	7	5	6	6	5	6	5	5	6	7	4	6	8	4	4	6	9	10	8	145	5.80
240	5	5	5	5	3	4	3	6	6	6	5	6	5	7	2	1	8	3	3	3	7	5	3	4	5	115	4.60
Total	81	81	81	82	85	83	87	84	83	81	81	81	82	81	85	87	82	85	85	81	81	81	86	87	84	2077	83.08

Note: Totals of Response Types may be higher than the number of participants in each group as some participants made multiple errors.
Note: RT = Response Type; see Table 7 for explanations for other codes.

Types of Methods

The types of methods that students used to complete each question were also examined. The categories of “not done”, “no written method”, “standard written method” and “alternative written method” were used as described in Chapter 3. Questions marked “no written method” had an answer but no method was shown at all or the question was simply rewritten horizontally with an answer. Standard written methods and alternative written methods were defined in Chapter 1, with extra clarification in Appendix G, but Figure 6 shows an example of each.

(a) (b)

Figure 6. Examples of (a) a standard written method and (b) an alternative written method to find the total of 0.99 + 0.70.

Whole numbers: Table 12 shows the frequency of methods for all whole number questions on every student’s test. That is, 161 participants multiplied by five whole number questions. More than half the participants in each group used a standard written method for any given question for whole numbers. Although this preference for a standard written method was equally high for both groups, the students in the NCX group had a stronger preference for an alternative written method than those in the CX group, with a moderately large effect size of 0.61. In turn the CX group had a greater preference for recording no written method at all.

Table 12
Method Type Preference, by Context - Whole Numbers

	NCX		CX	
	n = 400		*n* = 405	
Method	*f*	%	*f*	%
ND	0	0.00	0	0.00
NWM	50	12.50	66	16.30
SWM	250	62.50	250	61.73
AWM	100	25.00	89	21.98
Total	400	100.00	405	100.00

Note: ND = not done; NWM = no written method; SWM = standard written method; AWM = alternative written method.

Table 13 shows how many students chose one single method to complete all the whole number questions. In both groups around 65 per cent of the students chose just one method, whether it was to use no method at all or to use a standard or alternative written method. For example, 38 of the 80 participants in the NCX group used a standard written method to solve all whole number questions. The proportions for each method are fairly similar in the NCX and CX groups.

Table 13
Participants Choosing One Method to Solve all Problems - Whole Numbers

	NCX		CX	
	n = 80		*n* = 81	
Method	*f*	%	*f*	%
ND	0	0.00	0	0.00
NWM	4	5.00	8	9.88
SWM	38	47.50	37	45.68
AWM	10	12.50	9	11.11
Total	52	65.00	54	66.67

Note: ND = not done; NWM = no written method; SWM = standard written method; AWM = alternative written method.

Decimal fractions. Table 14 shows that a standard written method was used by more than half the students in each group to answer any given decimal fraction question. For other methods slightly greater differences between groups were found, although numbers were small. Taken together, these findings suggest that the NCX questions provoked different types of thinking to the CX questions. It is possible that the money context, because of its familiarity, made the CX students more inclined to solve the problems mentally.

Table 14
Method Type Preference, by Context - Decimal Fractions

	NCX $n = 2000$		CX $n = 2025$	
Method	*f*	%	*f*	%
ND	3	0.15	54	2.67
NWM	242	12.10	466	23.01
SWM	1400	70.00	1218	60.15
AWM	355	17.75	287	14.17
Total	2000	100.00	2025	100.00

Note: ND = not done; NWM = no written method; SWM = standard written method; AWM = alternative written method.

Table 15 shows how many students chose one single method to complete all of the decimal fraction questions. In the NCX group 67 per cent of the students chose a single method, whereas in the CX group around 52 per cent did. This finding suggests the CX students were more likely than the NCX students to switch methods depending on the question. The fact that 60 per cent of the NCX students chose a standard written method to answer every decimal fraction question provides further strength to the finding that this method was highly preferred by the NCX students.

Table 15
Participants Choosing One Method to Solve all Problems - Decimal Fractions

	NCX n = 80		CX n = 81	
Method	*f*	%	*f*	%
ND	0	0.00	0	0.00
NWM	1	1.25	9	11.11
SWM	48	60.00	31	38.27
AWM	5	6.25	2	2.47
Total	54	67.50	42	51.85

Note: ND = not done; NWM = no written method; SWM = standard written method; AWM = alternative written method.

Figures 7, 8 and 9 show the percentage of all students who chose a particular method for each decimal fraction question. Generally, the use of no written method increased over the course of the tests, the use of alternative written methods decreased and the use of a standard written method generally remained constant for the NCX group and decreased slightly for the CX group. This held true even if those students who did not complete all questions are removed from the analysis. The increases and decreases in method choice from one question to the next were approximately in the same proportion for both groups, although the absolute percentages were different.

Figure 7 shows that the use of no written method increased over the course of this part of the test. There was also about a 10 per cent difference in preference between the two groups and this preference remained fairly constant for each question. Students in the CX group displayed a greater preference for a standard written method for each question.

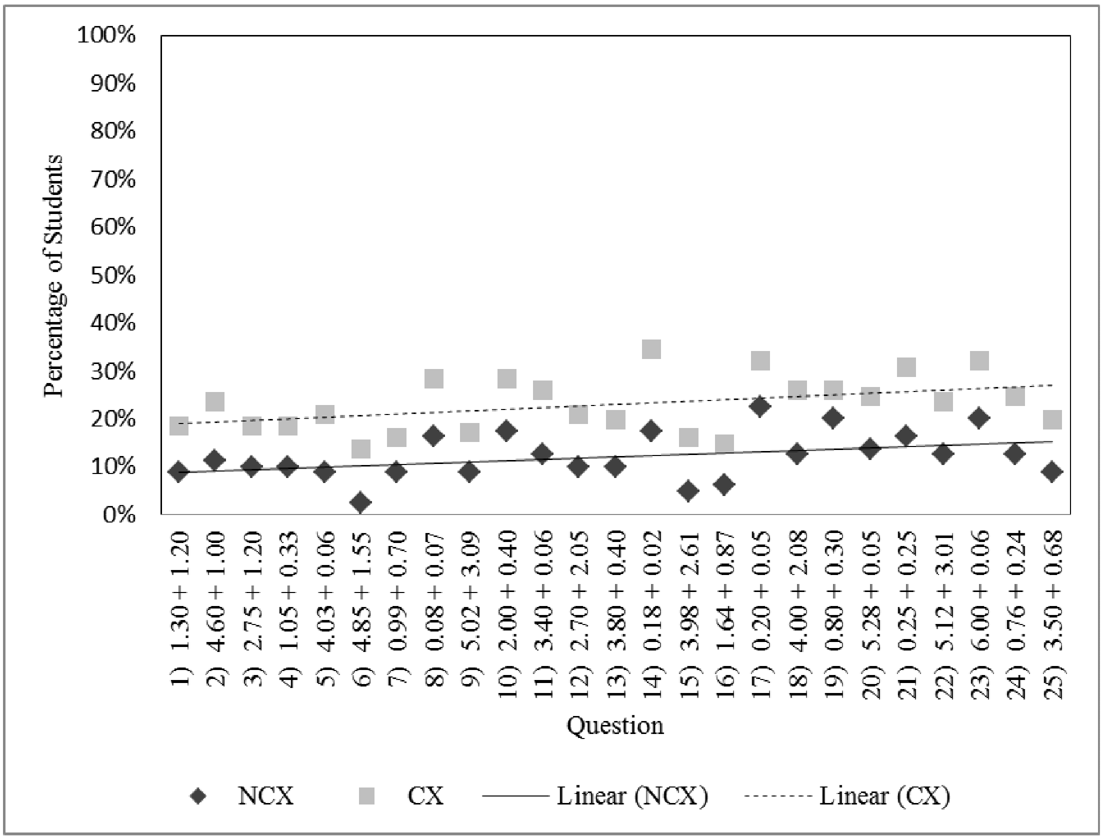


Figure 7. Percentage of participants per question recording no written method.

Figure 8 shows that the use of a standard written method was relatively steady throughout the test. The difference in frequency between the two groups for most questions was also around 10 per cent, as was the case for no written method. Students in the NCX group displayed a greater preference for a standard written method for each question.

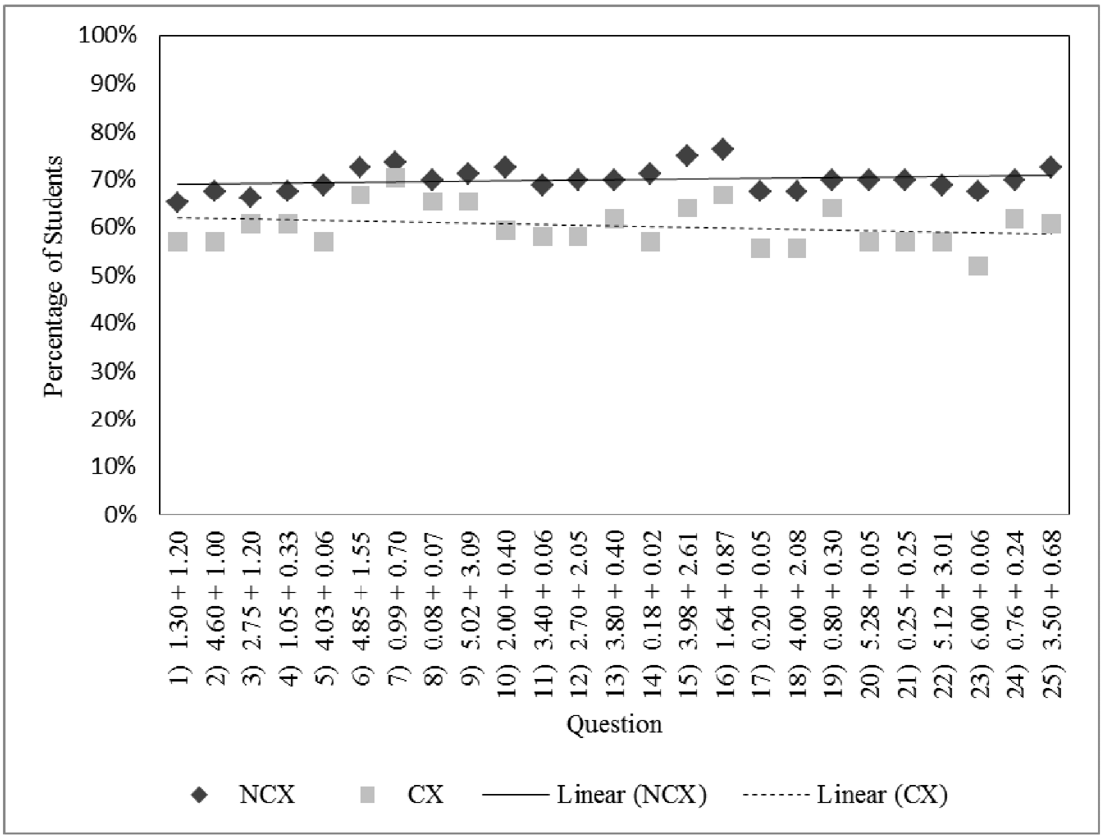


Figure 8. Percentage of participants per question recording a standard written method.

Figure 9 shows that student preference in both groups was very similar, with only a slight difference in preference between them. Generally the students in the NCX group displayed a greater preference for an alternative written method for each question.

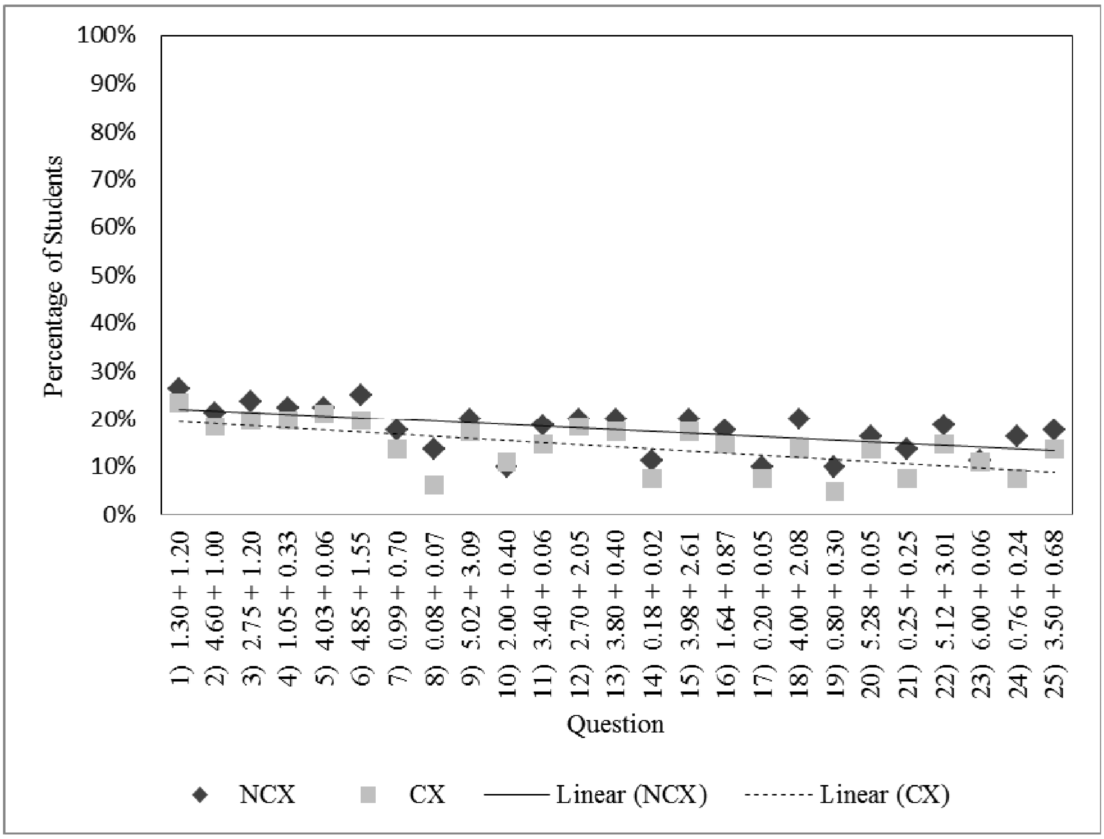


Figure 9. Percentage of participants per question recording an alternative written method.

One observation arising from analysis of methods is that some students used alternative or standard written methods for particular questions when it could be argued that none were necessary. For example, Question 8 (0.08 + 0.07) was a relatively easy question to answer based on the fact around 16 per cent of students in the NCX group and 28 per cent of students in the CX group were not using any written method at all to solve it. These were relatively high percentages for recording no written method at all. A possible reason that some students found it unnecessary to use any written method to calculate 0.08 + 0.07 is that it is the basic addition fact 8 + 7 but involving hundredths. Figure 10 shows the approaches that many other students took regardless.

(a) (b)

Figure 10. Use of (a) a standard written method, and (b) an alternative written method to find the total of 0.08 + 0.07.

Additional Data

Effects of year level. Table 16 shows that accuracy with whole number problems was slightly higher in Year 5 students compared to Year 4 for both the NCX and CX groups, as would be expected. The effect sizes, however, are at the lower end of what Hattie (2007) would describe as due to natural growth, indicating some slowdown in the Year 5 students' development.

Table 16
Effect of Year Level - Whole Numbers

	Year 4			Year 5			
Condition	*n*	*M*	*SD*	*n*	*M*	*SD*	*d*
NCX	43	4.26	1.03	37	4.49	0.90	-0.24
CX	43	4.51	0.83	38	4.68	0.57	-0.24

Similar findings were evident for the decimal fraction problems but to a greater degree, as shown in Table 17. These two findings together suggest that accuracy improves with age, although the small to moderate effect sizes suggest there was not a great deal of improvement. The mean scores reinforce this interpretation as there is a difference of only one to two marks between Year 4 and Year 5 scores for decimal fractions.

Table 17
Effect of Year Level - Decimal Fractions

	Year 4			Year 5			
Condition	*n*	*M*	*SD*	*n*	*M*	*SD*	*d*
NCX							
Scenario 1	43	21.51	3.88	37	22.95	3.67	-0.38
Scenario 2	43	21.51	3.88	37	22.95	3.67	-0.38
CX							
Scenario 1	43	19.84	6.85	38	21.39	4.48	-0.27
Scenario 2	43	21.12	5.24	38	22.47	3.01	-0.31

Considered together, the effect sizes suggest that there may be a curriculum effect. The effect sizes for decimal fraction problems indicate at least normal growth from Year 4 to Year 5, whereas the whole number results show there was less than expected growth between the year levels. This finding may be attributable to a greater emphasis by teachers on decimals and part whole numbers in Year 5, so that students had recent experience with decimal fraction addition.

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Effects of gender. Table 18 shows that females in both year levels had a very slight advantage in accuracy over males for the whole number problems.

Table 18
Effect of Gender - Whole Numbers

	Male			Female			
Year level	*n*	*M*	*SD*	*n*	*M*	*SD*	*d*
Year 4	37	4.30	0.88	49	4.45	0.98	-0.16
Year 5	39	4.56	0.79	36	4.61	0.73	-0.06
Both years	76	4.43	0.84	85	4.52	0.88	-0.10

Table 19 shows that Year 4 males performed slightly better than Year 4 females with decimal fraction problems, whereas Year 5 males performed slightly worse than Year 5 females.

Table 19
Effect of Gender - Decimal Fractions

	Male			Female			
Year level	*n*	*M*	*SD*	*n*	*M*	*SD*	*d*
Year 4							
Scenario 1	37	20.92	5.72	49	20.49	5.56	0.08
Scenario 2	37	21.43	4.65	49	21.22	4.59	0.05
Year 5							
Scenario 1	39	21.69	4.43	36	22.67	3.81	-0.24
Scenario 2	39	22.49	2.99	36	22.94	3.70	-0.14
Both years							
Scenario 1	76	21.32	5.08	85	21.41	4.99	-0.02
Scenario 2	76	21.97	3.90	85	21.95	4.30	0.01

There is no apparent explanation for this finding, although the difference in effect sizes between scenarios in Year 5 suggests that the male students were more careless with money notation than were the Year 5 females. Both year levels combined show that there was essentially no difference in accuracy between the genders.

Table 20 shows that females and males had considerable differences in preferences for methods with the whole number problems.

Table 20
Method Type Preference, by Sex - Whole Numbers

	Male n = 380		Female n = 425	
Method	*f*	%	*f*	%
ND	0	0.00	0	0.00
NWM	72	18.95	40	9.41
SWM	229	60.26	280	65.88
AWM	79	20.79	105	24.71
Total	380	100.00	425	100.00

Note: ND = not done; NWM = no written method; SWM = standard written method; AWM = alternative written method.

Table 21 shows that this difference in method preference carried through to the decimal fraction problems. Together with the whole number results, these findings suggest that most females felt more confident using a standard written method than an alternative written method when moving from whole number questions to decimal fraction ones.

Table 21
Method Type Preference, by Sex - Decimal Fractions

	Male $n = 1900$		Female $n = 2125$	
Method	*f*	%	*f*	%
ND	8	0.42	49	2.31
NWM	458	24.11	247	11.62
SWM	1114	58.63	1509	71.01
AWM	320	16.84	320	15.06
Total	1900	100.00	2125	100.00

Note: ND = not done; NWM = no written method; SWM = standard written method; AWM = alternative written method.

Summary

In this chapter several key results were identified. On the basis of accuracy with the whole number questions, the two groups were reasonably equivalent with no important differences between them in computational skills. Despite this, the NCX group performed the same as or better than the CX group with the decimal fraction questions, depending on the scenario. The results for the CX group reveal a high incidence of notation errors for particular questions, but also generally across the test, at a level that was not evident with the NCX group. There were also considerable differences between the groups for method choice, with the NCX group showing a greater preference for not recording a method, although both groups had a strong inclination for a standard written method overall. For the whole number and decimal fraction questions, females favoured a standard written method more than the males did.

In the next chapter the results are discussed, referring to research surveyed in Chapter 2 but also additional research that helps explain possible reasons for how the money context affected how answers were recorded. Also discussed is the choice of methods, both in selection according to context and in the effects of written methods generally.

CHAPTER 5: DISCUSSION

The previous chapters provided information about the use of money contexts in connection with decimal fraction addition. The results from testing 161 students on addition of decimal fractions where questions were either placed in a money context (CX) or were not contextually situated (NCX) revealed that there was little difference in accuracy but considerable differences in method choice.

In this chapter, the results are discussed in light of previous research and educational theory explored in Chapter 2. On the basis of the results, it appears that trying to contextualise decimal fractions involving hundredths with the use a dollar symbol does not give students any advantage in accuracy. Contextualising decimal fractions in this way, however, may have some influence on the methods students choose. The findings from the study will be discussed under the relevant research questions.

Research Question 1

How does accuracy with addition differ when decimal fractions to hundredths are written with dollar signs compared to when they are not?

At the year levels studied, results from this study showed that placing addition of decimal fractions in a money context had no positive effect on accuracy. One reason may be that any advantages that contextualised problems are argued to have (Beswick, 2011; Boaler, 1994; Putnam & Borko, 2000) are less likely with the limited context provided by including the dollar sign ($). It may be that placing each question in a word problem or providing a picture with a price tag would invoke stronger application of outside-school experience, with a subsequent increase in accuracy, though van den Heuvel-Panhuizen (2005) suggests that such devices essentially play decorative purposes rather than provide any cognitive benefit.

It was not the intention of this study to investigate different approaches to placing problems in context. Rather it was to investigate a common current practice.

The work of Steinle et al. (2006) suggested that money is not a good choice for developing conceptual understanding and procedural fluency with decimal fractions as the number of decimal places generally never varies. It is the case that sometimes money is reported to more than two decimal places in applications such as currency conversion. Steinle et al. (2006) noted that some adults end up truncating, mentally or physically, such decimal fractions after two decimal places with the assumption that the extra digits are errors or non-essential. Hence the potential for errors in money contexts would be greater. Seeing money with three or more decimal places may be sufficiently unusual that students' accuracy would suffer more than it would if they worked with the same decimal fractions in a context-free format.

A set of notation errors involving dollar and cent symbols arose in the CX group. Response Type 140 involved reporting amounts less than or equal to a dollar a whole number. Response Type 141 involved using no money symbols at all for amounts greater than a dollar. Response Type 240 involved reporting amounts less than a dollar without any money symbols or decimal notation, using the wrong money symbol, or using dollar symbols and cent symbols at the same time. Response Type 240 necessitated the creation of two scenarios for recording and analysing results. In an ordinary classroom setting, this particular type of response may present problems for assessment. The students may have simply recorded an answer without using the correct money symbols or they may have deeper conceptual misunderstandings involving money, decimal fractions or both.

Using different contexts, such as measurement, might reduce the errors associated with symbol notation in Year 4 and Year 5 because the context is less frequently experienced. With money it could be reasonably expected that most students have had many experiences in school and non-school contexts by Years 4 and 5, and consequently have become accustomed to thinking of the relationship between dollars and cents. Van den Heuvel-Panhuizen (2005) argued that students' familiarity with context can hinder finding an answer as they take into account extraneous "real-life" knowledge of the context. Thus, using a non-money unit might circumvent students' strong associations with money in everyday contexts. However, if contexts need to be unfamiliar enough to avoid students' existing outside-school knowledge interfering, perhaps the use of context in this manner serves no real purpose.

Aside from notation discrepancies, the results suggested that for some students emphasising "real-life" contexts may be detrimental, as noted by Boaler (1994) and van den Heuval-Panhuizen (2005). The everyday knowledge of money in Australian contexts appeared to have been used by four students who rounded the amounts either before or after addition. Although the students who rounded after addition generally applied their real-world knowledge of how monetary amounts are rounded in Australia accurately, the students who rounded before addition missed the subtleties of rounding which meant that their answers were incorrect. It was expected that more students would use rounding, especially given the emphasis on transactions in the Queensland and Australian curricula. It may be that rounding meant little to students at this stage of their lives. Older students working in jobs requiring rounding or who are more familiar with rounding from using their own money regularly might round more often if they completed similar tests to those used in this study.

Aside from money notation errors, most other errors for the NCX and CX groups were because of incorrect addition (e.g. 6 + 7 = 11), transcribing a question incorrectly before addition (e.g. writing 0.06 as 0.60) or because of a mechanical error in the method used (e.g. forgetting a carry digit). These types of errors were generally also the most frequent ones for the whole number component of the tests. The mechanical errors varied in type and level of seriousness but centred on the application of a written method, not an understanding of place value as such. As the overall inaccuracy rate for NCX and CX groups was between 11 and

16 per cent (depending on scenario) the results for this study suggest that most Year 4 and Year 5 students in Queensland in the second half of the school year can add decimal fractions accurately.

Generally, a number of errors were identified in answers for all decimal fraction questions. Of particular interest are the questions that had the highest number of errors for each group. For both the NCX and the CX groups, Questions 6, 7, 8, 14, 15, 16 and 25 were among those that had the highest frequency of errors. The questions are shown in Table 11, in Chapter 4. All those questions involved regrouping to various degrees so it seems likely that regrouping triggered confusion at some level, whether in basic addition fact recall or a method for working through the regrouping process. Perhaps for Question 8 and Question 14, some conceptual difficulty with place value also occurred as the questions involved addends that had digits only in the hundredths place. Also noticeable was the fact that Questions 8, 14, 21 and 23 had the highest frequency of any Response Type (other than 110). Response Type 140, which involved reporting amounts less than a dollar as cents, was displayed by roughly one-quarter of all CX participants for each question. It seems likely that those students were applying knowledge that was grounded very firmly in what those amounts meant in a money context, a case of bringing additional "real-life" detail into a problem (Boaler, 1994; van den Heuval-Panhuizen, 2005).

Research Question 2

How do addition methods differ when decimal fraction problems using hundredths are contextualised with dollar signs compared to when they are not?

Although there was little difference in accuracy between groups, the results suggested that context influenced the choice of methods when adding decimal fractions. The NCX group and CX group had very similar preferences in methods for solving the whole number problems. For decimal fractions, the general pattern of method choice over the test papers was also similar. The CX group, however, generally showed a greater preference than the NCX group for not using a written method at all.

Thompson (1994) found that in some classrooms where standard algorithms were not taught around three-quarters of the 9- to 10-year old students preferentially set out their work in a horizontal format (though perhaps this too was an effect of teacher-demonstrated strategies that purposely avoided vertical formats). Many of the alternative written methods shown by NCX and CX students were set out in a horizontal format although some had non-standard vertical formats. This contrasts strongly with the space provided in textbooks available in Queensland and described in Chapter 2. As can be seen in the examples in Appendix A, the format for answering questions in many cases promotes a standard written method or otherwise prevents students from using an alternative written method in the space provided. Given the findings from this study it seems that this might unnecessarily hamper student decision-making as to which method would be best to use for any given question.

A greater proportion of NCX students used a single method to complete all decimal fraction questions than did CX students. Perhaps the greater familiarity of money made the CX students not only more confident with their mental abilities to solve the problems but also more comfortable in changing methods to suit a given question. Verschaffel, Luwel, Torbeyns and Van Dooren (2009, p. 338) referred to this phenomenon as “adaptivity”. They contrasted this ability with “flexibility” which they argued was the use of multiple strategies, the implication being that adaptive students can use and appropriately choose strategies, while flexible students can use a range of strategies but not necessarily choose the one most suited to a given task (Verschaffel et al., 2009). The current version of the Australian Curriculum (ACARA, 2012) seems to position both adaptivity and flexibility as attributes of “fluency”, one of the four sets of actions or behaviours that students should demonstrate. Because contextualising decimal fractions with a dollar symbol may provoke greater adaptivity in students than when the fractions are non-contextualised, using such a context would help meet the aims of the Australian Curriculum. This study suggests, however, that the adaptivity may only occur with contextualised decimal fractions, not with non-contextualised ones. This may mean more discussion is necessary in classrooms to highlight the similarities between the two. Care would need to be taken not to over-emphasise the money context so that students do not rely on it as a model of all decimal fractions, but just as a limited example (Steinle et al., 2006).

Although there were distinct differences in the choice of methods between the NCX and CX groups, it would be hard to make firm conclusions about particular types of questions prompting particular methods. Figures 7, 8 and 9 in Chapter 4 illustrate how the use of particular methods increased or decreased over the course of the test in approximately the same proportions for both groups. It is possible that this change in method over the course of the test was because of test effects: namely, that at some point during the test students either out of necessity (because of time restraints) or preference realised that many of the problems could be solved without recording a method.

For example, Question 1 (1.30 + 1.20) was chosen by the researcher as the first decimal fraction question on the tests because of its relative simplicity. It had one of the lowest frequencies for using no written method (NCX 9%; CX 19%). A later question was Question 18 (4.00 + 2.08) which had a higher frequency for no written methods (NCX 13%; CX 26%), and a similar frequency as Question 1 for the use of standard written method.

Another example is found with Question 5 (4.03 + 0.06) and Question 23 (6.00 + 0.06). No written methods were used for Question 5 by 9 per cent of students in the NCX group and 21 per cent of students in the CX group. For Question 23 the frequency was 20 per cent (NCX) and 32 per cent (CX). The use of a standard written method was also roughly the same for both questions. Similar discrepancies can be found with other pairs of questions that are approximately equivalent in regards to the absence of regrouping. These examples suggest that some change took place during the tests that was not solely because of the attributes of the questions. Adaptivity may have been evident for some students, but it may not be the only reason methods changed.

Standard written methods, it is sometimes argued, discourage thinking (Morgan, 2000; Reys, 1985). The design of standard written methods means that, once mastered, the algorithms can be used for any applicable situation with little regard for the nature of the numbers being operated on (Kamii & Dominick,1998; Morgan 2000). Although this is seen as a strength by some (Klein & Milgram, 2000), others argue that algorithms "contribute little to the development of number sense, particularly where decontextualised examples are presented to students" (Morgan, 2000, p. 7). Alternative written methods have been proposed that are meant to demonstrate and foster mental computation (Baker & Baker, 2007; Kennedy, Tipps & Johnson, 2011). The results from this study suggest that some of these alternative methods also suffer the fate, like standard algorithms, of being used by students "with little sense as to why, how, or what they are doing" (Reys, 1985, p. 44). This was evident in some students' persistent use of particular alternative written methods despite easier solution paths being applicable. Heinze, Marschick and Lipowsky (2009) made a similar comment about students having a favourite method to solve all problems, regardless of the question's attributes. The methods in Figure 11, replicated from three participants' work, illustrate this point. It should be noted that while some methods (such as (a) in Figure 11) recorded sub-totals that were mathematically inaccurate regarding the meaning of the digits, students were nonetheless able to work out how to translate the steps to attain the correct answer.

0.08 + 0.07 = 0.15

0 + 0 = 0
0 + 0 = 0
7 + 8 = 15
0 + 0 + 15 = 0.15

(a)

5.28 + 0.05 = 5.33

0.08+0.02=0.10, 0.10+0.03=0.13, 0.13+0.20=0.33, 0.33+5.00=5.33

(b)

Figure 11. Two alternative written methods.

Perhaps the reason for the use of these methods is similar to what Hiebert and Wearne (1985) asserted takes place with the standard written method. Namely, that placing the addends in a familiar format reassures the student that the problem is ready to be solved and triggers the rules needed to solve it. In the method shown in (a) of Figure 11, writing the totals for adding the zeros in the ones and tenths place seems unnecessary. In the method shown in (b), instead of progressively adding each place, the five hundredths in 0.05 could have been more efficiently added on to the whole amount of 5.28. So although for some questions it seems unnecessary to use a written method, for the students who did use one, working methodically to add each place using a familiar method might have acted as reassurance that the correct total would be arrived at in due course.

Related to an “unthinking” application of written methods, preference for standard or alternative written methods may be over-reported in this study. It was observed during the administration of the tests that some students wrote their reported answer first then wrote out a standard written method. It was also noticed that some students switched completely from using one method more or less consistently, to using another method to complete the remainder of the test. As Reys (1984, p. 550) noted, “Students are often chastised for not showing all their work, so they may record a written algorithm even though it is unnecessary”, thus discouraging mental computation and the use of any relationships between the numbers that would make the calculation easier. It was certainly noticed that a money context introduced errors in notation for the CX group. The common request in classrooms to “show your thinking/working” may have introduced other errors for both CX and NCX students.

Additional data

Transfer. One intriguing possibility raised by this study is that the NCX participants used existing knowledge of money to help them answer the “bare-number problems”. Some students gave direct written explanations stating they did use such knowledge (“I thought of it like money”). Although these statements occurred infrequently, it may be that the actual prevalence of this type of thinking was much higher. It is also possible that other students thought of CX problems as simply involving ordinary decimal fractions with an extra symbol to record. About 7 per cent of Response Types for the CX group involved students not using any money notation symbols at all (Response Type 141), so it appears that at least for some students the money context may have been irrelevant. This might be an example of what Beswick (2011, p. 387) described as “unspoken rules concerning which elements of context problems should be attended to and which can safely be ignored”.

The types of responses that students made may be useful in interpreting whether students in the NCX group used any existing knowledge of money to help them. The three Response Types most linked to money notation are 030, 140 and 240. Response Type 030 involves rounding addends to a multiple of five before addition; a misapplication of the rounding of cash totals that occurs in Australia. Response Type 140 involves reporting “$1.00” as “$1” or amounts such as “$0.50” as “50c”. Response Type 240 involves reporting amounts such as “$0.50” as “0.50c” or “50”. It could be expected that Response Type 030 would not occur often as it was very infrequent in the CX group (less than 1 per cent of Response Types), and that Response Type 140 would be unlikely to occur in the NCX group because it would mean introducing extra symbols on to the page. But it could be argued that Response Type 240 would be quite likely if students in the NCX group were thinking of numerals such as “0.50” as 50 cents and then record the answer as “50”. However, the results showed that only one student in the NCX student made responses of this type and this student had actually rewritten most of the questions as money. In comparison, almost 6 per cent of all Response Types for the CX group were Response Type 240. If NCX students were thinking of the questions as dollar-and-cent amounts they appeared to have avoided recording their thinking about amounts less than a dollar in ways that the CX group did.

Another indicator of whether NCX students were thinking of money is to examine the methods used. Assuming that the students in both groups were equally familiar with money it was surprising that the reliance of the NCX group on a standard written method was greater than that of the CX students. An explanation for the differences in choice of method and the manner in which answers were reported could lie in how decimal fractions are conceptualised by the students. It may be that the connection between money and decimal fractions is never made explicit except in a superficial way (“They look the same”), so that the students in effect end up working with different sets of numbers: decimal fractions and money. Instead of contextualised problems promoting mental computation it could be that the links are missing between money and decimal fractions. In other words, the students have not transferred their knowledge of working with money to working with more abstract forms of decimal fractions, a danger noted by Anderson, Reder and Simon (2000).

Age. The results between year levels suggest that accuracy improves from one year to the next for both groups. This is to be expected for a number of reasons, such as Year 5 students having had extra practise with basic addition facts, longer use of addition methods, and greater exposure to decimal fractions in keeping with the expectations given in the curriculum documents described in Table 1 in Chapter 2. The effect size for age regarding accuracy was found to be 0.24. Hattie (2009) argued that an effect size between 0.20 and 0.40 constitutes “average” growth per year so the effect size form this study is in line with Hattie’s analysis.

Gender. There were differences in the choice of method between males and females, which was an unexpected finding. A greater proportion of females than males had a preference for using a standard written method for whole numbers and decimal fractions. Investigating the reasons for this is beyond the scope of this study, but it is an intriguing result. Carr, Jessup and Fuller (1999) suggested that parent and teacher perceptions of boys’ and girls’ abilities in mathematics may influence the types of strategies that each are encouraged to use. In particular, they argued that boys’ may be given more freedom than girls to explore unconventional methods. In this study, this could account for the greater use of alternative written methods or not recording any written methods by males compared to females. Perhaps an unintended effect of this was that there was some small difference in accuracy in favour of Year 5 females compared to the Year 5 males, so perhaps the high use of a standard written method has an increased effect on accuracy.

Limitations

There are several limitations in a study of this size and scope that impact on the interpretation of the results. The tests did show a slight ceiling effect, with a majority of students answering most questions correctly. This study aimed to identify issues around current practice and it was not the intention to measure the limits of achievement. Hence the tests were designed so that they matched curriculum expectations and provided a variety of types of response. The aim was to keep the part-whole structure of the numbers as simple as possible while still including hundredths.

Schools were chosen for the study on the basis of NAPLAN results with the intention of identifying average performing schools. For practical reasons, travel time and willingness to participate also became important criteria and these situations may have led to a slightly less representative sample. Given the care with which the schools were chosen, however, it is likely that this consideration did not impact greatly on the study. The total number of Queensland full-time students in 2010 in Year 4 was about 39 000 and in Year 5 was about 57 000 (ABS, 2012). Testing took place during August and September of 2011 in the third term of the school year. During this period, the mean age of the Year 4 students in the study was 9.0 (Queensland 2011 full-time student mean 8.8) and the mean age of the Year 5 students was 9.8 (Queensland 2011 full-time student mean 9.7) showing that students in the study were representative in terms of age (ABS, 2011). The Queensland mean score on the Year 3 Numeracy section of the 2010 NAPLAN test was 378.5 (*SD* = 65.9) (QSA, 2010b). The mean for the schools involved in this study was 388.9 (*SD* = 65.9). Regarding achievement, the schools in the study were reasonably representative of the central tendency of the wider school population.

An explicit instruction to students to record a method only if they found it necessary to do so might have revealed different reporting of what methods students used. The instruction to record their thinking may have literally forced their hand in the type of method to use, and consequently also introduced various procedural and notation errors affecting accuracy. If the nature of the methods used was to be considered, however, it was necessary to have examples of the students' working. Students are accustomed to this instruction in their classrooms.

The whole number questions on the tests were used to provide some measure of equivalence between groups. Additional whole number questions may have helped to develop a stronger means of establishing pre-test equivalence between groups. The focus of the study, however, was on decimal fractions and because of time restrictions on the test used, whole number questions were minimised.

The test instruments did not appear to cause any problems regarding physical layout or clarity of questions, and generally there was sufficient time allowed for the vast majority of students to complete them. Although the tests did provide the information required, further analysis and possible adjustment of the test instruments is warranted.

Future research

The results of this study indicate that contextualising decimal fraction addition in a money context had no greater effect on accuracy than without context. It also introduced notation errors and had some influence on choice of method. It would be advantageous to explore whether these results would occur with the same questions but in other contexts. For example, length and mass are two contexts where Year 4 and Year 5 students' lesser familiarity with the units might cause noticeable differences in accuracy and method choice compared to what was found in this study. Using contexts such as metric length and metric mass would also allow "ragged" decimals (e.g. 2.5 + 1.372) to be investigated, which the money context in this study could not allow. It would be expected that an increased rate of procedural errors would occur, but it might also highlight differences in method selection between contextualised problems and non-contextualised problems.

Replicating the study using the same test questions but reordered would help to clarify whether the choice between no written method and an alternative written method which developed to some extent across the tests occurs because of test effects or question structure. Increasing the age range to include Years 8 or 9 could show whether an increased knowledge and use of money means that there is also more rounding of answers to the nearest five cents or other forms of contextual interference.

For teachers, curriculum designers and assessors, the results indicate that some degree of care is needed in contextualising problems with the simple inclusion of a dollar symbol. It is not obvious that students in Years 4 and 5 make links between contextualised and non-contextualised decimal fractions, at least not when the only difference is a dollar symbol. Possibly more explicit discussion of the relationship between money and non-contextualised decimal fractions needs to occur to take advantage of any benefit that context may offer. Further, caution must be used when marking problems placed in a money context to ensure that conceptual errors, either with money and/or decimal fractions generally, are not misconstrued as simple notation errors.

Regarding the choice of method for solving addition with decimal fractions (whether contextualised or not), the results suggest that when given free choice many students preferred to use a method other than a standard written method. The standard method was still the most popular choice, but not the only choice, and greater consideration of student preference for solution methods might be beneficial, a point noted by McIntosh (2005a). Unless students are being taught specifically a particular written method, the question may be better presented as a horizontal number sentence so that students can then choose a method that seems most suitable. The fact that alternative and standard written methods were also used for some questions when neither were really required, points to the important place that discussion of method choice should play in classrooms.

Conclusion

Some of the research literature suggests that using money as a context has little positive effect on the conceptual understanding of decimal fractions. The results from this study also suggest that contextualising addition with the inclusion of a dollar symbol may have no positive effects on accuracy for Queensland students in Years 4 and 5. It does appear, however, that using money contexts could prompt students to use a mental method or alternative written method to calculate the answer more than they would without that context. This thinking may be influenced by gender in older students. Contextualising with a money context like that used in this study may also generate extra difficulties for students and teachers in how calculations are recorded and assessed. Perhaps it is time to question the accepted wisdom that contextualised problems are simpler and should be taught before non-contextualised ones. This study suggests that more effort is needed to help students robustly apply methods for the addition of both types of decimal fractions, and to properly understand money notation. Only then can students confidently deal with the contextualised and non-contextualised addition of decimal fractions that is required outside as well as inside their mathematics classroom.

APPENDIX A: SAMPLES FROM TEXTBOOKS

GO Maths Level 3A (Year 4) (Burnett & Irons, 2007a, p. 112)

Loop together coins that show whole dollars or add up to whole dollars.
Then complete the sentences.

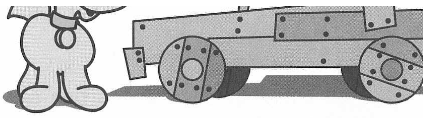
c.

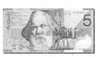
_____ dollars and _____ cents left over

The total is $ ____________.

iMaths 4 (Linthorne et al., 2010a, p. 54)

2 Use the least number of coins and notes possible to give change for each purchase. Calculate the total then write the letters in the correct money bags below.



Change from		if the cost is $3.80		=		E
Change from		if the cost is $1.40		=		T
Change from		if the cost is $5.90		=		D
Change from		if the cost is $3.30		=		I

Maths Plus 4 (O'Brien & Purcell, 2009a, p. 140)

1 Add or subtract these amounts.

a	b	c	d	e
$3.66	$4.54	$7.45	$6.73	$6.74
+ $2.06	+ $3.61	− $6.23	− $3.26	+ $2.64
____	____	____	____	____

New Signpost Maths, Student Book 4 (McSeveny et al., 2009a, p. 16)

4

a	b	c	d	e
$43.78	$72.75	$51.82	$64.24	$35.51
+$25.93	+$26.27	+$37.36	+$13.57	+$21.98

Targeting Maths, Year 4 (Turner, 2008, p. 99)

3

a	b	c	d	e
6 • 2 1	4 • 9 3	8 • 0 2	3 • 8 9	5 • 1 4
+ 4 • 3 8	+ 6 • 0 8	+ 1 • 9 4	+ 1 • 0 1	+ 4 • 6 3
____	____	____	____	____

GO Maths Level 3B (Year 5) (Burnett & Irons, 2007b, p. 119)

Draw the extra coins you would need to pay these prices.

iMaths 5 (Linthorne et al., 2010b, p. 70)

Write each sum vertically and add.

a 7.05 + 1.3 + 8.14 (estimate)

b 3.02 + 0.5 + 4.19 (estimate)

c 22.43 + 17.09 (estimate)

d 9.47 + 9.47 (estimate)

Maths Plus 5 (O'Brien & Purcell, 2009b, p. 116)

2 Calculate the total price of each shopping docket and then round it to the nearest 5 cents.

a

Comic book	$1.23
Pencil	$0.28
Maths book	$10.92
Pen	$1.37
Ruler	$0.48
Total	$
Rounded	$

b

Pen	$1.37
Ruler	$0.48
Ruler	$0.48
Ruler	$0.48
Comic book	$1.23
Total	$
Rounded	$

c

Maths book	$10.92
Comic book	$1.23
Pen	$1.37
Pen	$1.37
Ruler	$0.48
Total	$
Rounded	$

d

Maths book	$10.92
Eraser	$0.32
Comic book	$1.23
Comic book	$1.23
Comic book	$1.23
Total	$
Rounded	$

New Signpost Maths, Student Book 5 (McSeveny et al., 2009b, p. 67)

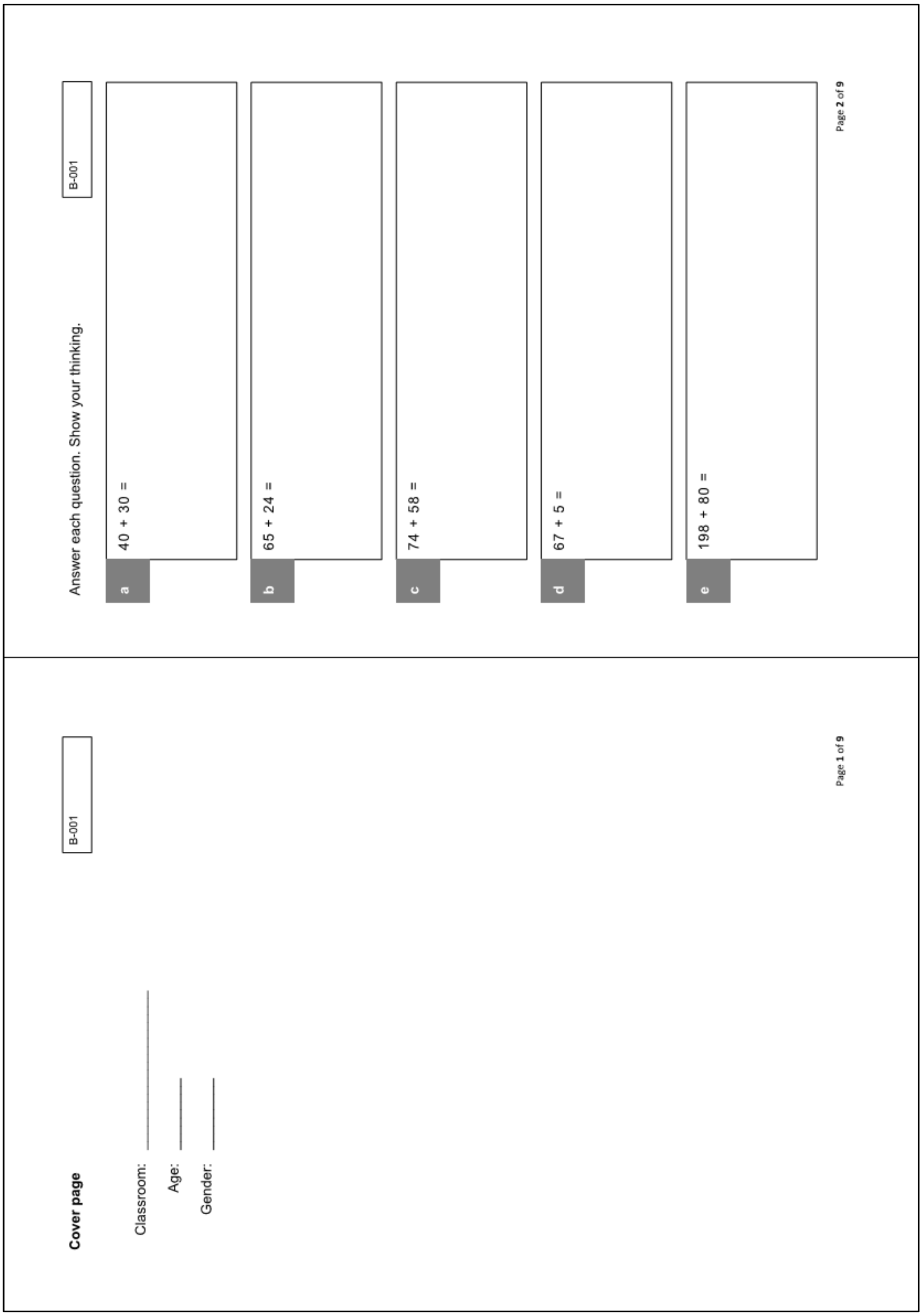
f	g	h	i	j
35·63	47·5	23·19	28·86	46·86
19·8	32·48	26·74	39·75	27·7
+12·13	+10·65	+19·6	+ 7·03	+14·62

Targeting Maths, Year 5 (Turner, 2010, p. 22)

a $	c	b $	c	c $	c	d $	c
0 .	10	3 .	40	1 .	15	3 .	25
+ 2 .	50	+ 2 .	30	+ 2 .	25	+ 4 .	25

APPENDIX B: CX TEST

Each page of the test was printed to fit A4 paper (i.e. approximately 21 cm by 30 cm).



Cover page

Classroom: ____________

Age: ______

Gender: ______

B-001

Page 1 of 9

Answer each question. Show your thinking.

B-001

a 40 + 30 =

b 65 + 24 =

c 74 + 58 =

d 67 + 5 =

e 198 + 80 =

Page 2 of 9

Answer each question. Show your thinking.

B-001

1. \$1.30 + \$1.20 =

2. \$4.60 + \$1.00 =

3. \$2.75 + \$1.20 =

4. \$1.05 + \$0.33 =

Answer each question. Show your thinking.

B-001

5. \$4.03 + \$0.06 =

6. \$4.85 + \$1.55 =

7. \$0.99 + \$0.70 =

8. \$0.08 + \$0.07 =

Answer each question. Show your thinking.

B-001

9 $5.02 + $3.09 =

10 $2.00 + $0.40 =

11 $3.40 + $0.06 =

12 $2.70 + $2.05 =

Answer each question. Show your thinking.

B-001

13 $3.80 + $0.40 =

14 $0.18 + $0.02 =

15 $3.98 + $2.61 =

16 $1.64 + $0.87 =

Answer each question. Show your thinking.

B-001

17 $\$0.20 + \$0.05 =$

18 $\$4.00 + \$2.08 =$

19 $\$0.80 + \$0.30 =$

20 $\$5.28 + \$0.05 =$

Answer each question. Show your thinking.

B-001

21 $\$0.25 + \$0.25 =$

22 $\$5.12 + \$3.01 =$

23 $\$6.00 + \$0.06 =$

24 $\$0.76 + \$0.24 =$

Answer each question. Show your thinking.

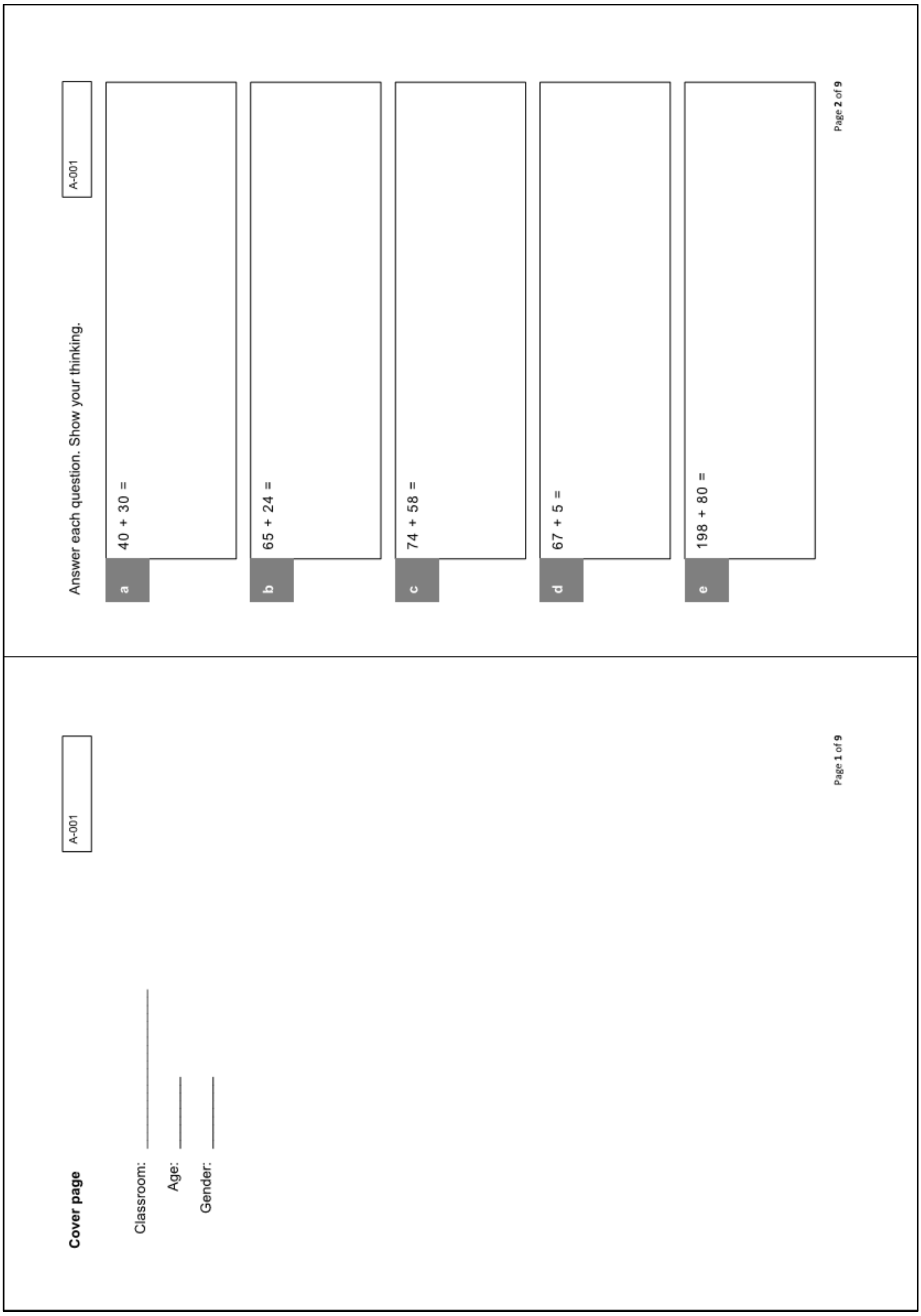
B-001

25 $\$3.50 + \$0.68 =$

FINISH

APPENDIX C: NCX TEST

Each page of the test was printed to fit A4 paper (i.e. approximately 21 cm by 30 cm).



Cover page

A-001

Classroom: ____________

Age: ______

Gender: ______

Page 1 of 9

Answer each question. Show your thinking.

A-001

a 40 + 30 =

b 65 + 24 =

c 74 + 58 =

d 67 + 5 =

e 198 + 80 =

Page 2 of 9

Answer each question. Show your thinking.

A-001

1 $1.30 + 1.20 =$

2 $4.60 + 1.00 =$

3 $2.75 + 1.20 =$

4 $1.05 + 0.33 =$

Answer each question. Show your thinking.

A-001

5 $4.03 + 0.06 =$

6 $4.85 + 1.55 =$

7 $0.99 + 0.70 =$

8 $0.08 + 0.07 =$

Answer each question. Show your thinking.

A-001

9 $5.02 + 3.09 =$

10 $2.00 + 0.40 =$

11 $3.40 + 0.06 =$

12 $2.70 + 2.05 =$

Answer each question. Show your thinking.

A-001

13 $3.80 + 0.40 =$

14 $0.18 + 0.02 =$

15 $3.98 + 2.61 =$

16 $1.64 + 0.87 =$

Answer each question. Show your thinking.

A-001

17 $0.20 + 0.05 =$

18 $4.00 + 2.08 =$

19 $0.80 + 0.30 =$

20 $5.28 + 0.05 =$

Answer each question. Show your thinking.

A-001

21 $0.25 + 0.25 =$

22 $5.12 + 3.01 =$

23 $6.00 + 0.06 =$

24 $0.76 + 0.24 =$

Answer each question. Show your thinking.

A-001

25 $3.50 + 0.68 =$

FINISH

APPENDIX D: CREATION OF THE TESTS

Figure D.1 shows the expressions that are possible when combining two addends comprised of ones (O), tenths (t) and hundredths (h). The use of the letter for a place indicates a non-zero digit in that place value column; the absence of a letter indicates a zero is used. For example, O,t,h + t represents an expression such as 1.55 + 0.30. There is a line of symmetry through the table, running diagonally from O + O down to O,t,h + O,t,h. The expressions on either side of this line are reflections of each other because of the commutative property of addition. Thus the expressions on one side of the line (shaded grey) were discarded as superfluous.

	O	**t**	**h**	**O,t**	**O,h**	**t,h**	**O,t,h**
O	O + O	O + t	O + h	O + O,t	O + O,h	O + t,h	O + O,t,h
t	t + O	t + t	t + h	t + O,t	t + O,h	t + t,h	t + O,t,h
h	h + O	h + t	h + h	h + O,t	h + O,h	h + t,h	h + O,t,h
O,t	O,t + O	O,t + t	O,t + h	O,t + O,t	O,t + O,h	O,t + t,h	O,t + O,t,h
O,h	O,h + O	O,h + t	O,h + h	O,h + O,t	O,h + O,h	O,h + t,h	O,h + O,t,h
t,h	t,h + O	t,h + t	t,h + h	t,h + O,t	t,h + O,h	t,h + t,h	t,h + O,t,h
O,t,h	O,t,h + O	O,t,h + t	O,t,h + h	O,t,h + O,t	O,t,h + O,h	O,t,h + t,h	O,t,h + O,t,h

Figure D.1. Combinations of ones (O), tenths (t) and hundredths (h), where a letter represents a non-zero digit.

Twenty-eight basic expressions were left. These were then combined with different types of regrouping. Regrouping occurs when the sum for any given place is larger than 9, resulting in an increase of 1 in the next place to the left. For example, adding the digits in the hundredths place in the expression 1.58 + 1.37 results in 15 hundredths. This total is then regrouped into 1 tenth and 5 hundredths, with the extra 1 tenth being then added to the existing 5 tenths and 3 tenths. Table D.1 shows the result of combining the 28 expressions identified above with different types of regrouping. Sample numbers are used in Table D.1 to illustrate the results.

Table D.1
Types of Regrouping

	Types of regrouping			
Basic expression	No regrouping	Regroup tenths	Regroup hundredths	Regroup tenths and hundredths
O + O	1.00 + 1.00	n/a	n/a	n/a
t + O	0.50 + 1.00	n/a	n/a	n/a
t + t	0.50 + 0.30	0.50 + 0.70	n/a	n/a
h + O	0.05 + 1.00	n/a	n/a	n/a
h + t	0.05 + 0.50	n/a	n/a	n/a
h + h	0.05 + 0.03	n/a	0.05 + 0.07	n/a
O,t + O	1.50 + 1.00	n/a	n/a	n/a
O,t + t	1.50 + 0.30	1.50 + 0.70	n/a	n/a
O,t + h	1.50 + 0.05	n/a	n/a	n/a
O,t + O,t	1.50 + 1.30	1.50 + 1.70	n/a	n/a
O,h + O	1.05 + 1.00	n/a	n/a	n/a
O,h + t	1.05 + 0.50	n/a	n/a	n/a
O,h + h	1.05 + 0.03	n/a	1.05 + 0.07	n/a
O,h + O,t	1.05 + 1.50	n/a	n/a	n/a
O,h + O,h	1.05 + 1.03	n/a	1.05 + 1.07	n/a
t,h + O	0.55 + 1.00	n/a	n/a	n/a
t,h + t	0.55 + 0.30	0.55 + 0.70	n/a	n/a
t,h + h	0.55 + 0.03	n/a	0.55 + 0.07	n/a
t,h + O,t	0.55 + 1.30	0.55 + 1.70	n/a	n/a
t,h + O,h	0.55 + 1.03	n/a	0.55 + 1.07	n/a
t,h + t,h	0.55 + 0.33	0.55 + 0.73	0.55 + 0.37	0.55 + 0.77
O,t,h + O	1.55 + 1.00	n/a	n/a	n/a

Table D.1 (Continued)

O,t,h + t	1.55 + 0.30	1.55 + 0.70	n/a	n/a
O,t,h + h	1.55 + 0.03	n/a	1.55 + 0.07	n/a
O,t,h + O,t	1.55 + 1.30	1.55 + 1.70	n/a	n/a
O,t,h + O,h	1.55 + 1.03	n/a	1.55 + 1.07	n/a
O,t,h + t,h	1.55 + 0.33	1.55 + 0.73	1.55 + 0.37	1.55 + 0.77
O,t,h + O,t,h	1.55 + 1.33	1.55 + 1.73	1.55 + 1.37	1.55 + 1.77

Note: O = ones; t = tenths; h = hundredths; n/a = not applicable

Combining the different types of regrouping with the expressions identified previously gave an additional 23 possible basic expressions. Of these 51 basic expressions, 25 were chosen to represent the basic expressions and the regrouping possibilities. Specific addends were then chosen to cover expressions that were deemed as “realistic”, “slightly unrealistic” or “very unrealistic” in everyday money contexts, along with ones that may prompt adjustment of the numbers before addition or trigger “rounding” of totals learnt from real-life money transactions. The final expressions are shown in Table D.2.

Table D.2
Questions Chosen for NCX and CX Tests

	Types of regrouping			
Basic expression	No regrouping	Regroup tenths	Regroup hundredths	Regroup tenths and hundredths
O + O	not chosen	n/a	n/a	n/a
t + O	2.00 + 0.40[b]	n/a	n/a	n/a
t + t	not chosen	0.80 + 0.30[b]	n/a	n/a
h + O	6.00 + 0.06[ce]	n/a	n/a	n/a
h + t	0.20 + 0.05[c]	n/a	n/a	n/a
h + h	not chosen	n/a	0.08 + 0.07[c]	n/a
O,t + O	4.60 + 1.00[a]	n/a	n/a	n/a
O,t + t	not chosen	3.80 + 0.40[b]	n/a	n/a

Table D.2 (Continued)

O,t + h	3.40 + 0.06[ce]	n/a	n/a	n/a
O,t + O,t	1.30 + 1.20[a]	not chosen	n/a	n/a
O,h + O	4.03 + 2.08[ae]	n/a	n/a	n/a
O,h + t	not chosen	n/a	n/a	n/a
O,h + h	4.03 + 0.06[ce]	n/a	not chosen	n/a
O,h + O,t	2.70 + 2.05[a]	n/a	n/a	n/a
O,h + O,h	not chosen	n/a	5.02 + 3.09[ade]	n/a
t,h + O	not chosen	n/a	n/a	n/a
t,h + t	not chosen	0.99 + 0.70[bde]	n/a	n/a
t,h + h	not chosen	n/a	0.18 + 0.02[c]	n/a
t,h + O,t	not chosen	3.50 + 0.68[be]	n/a	n/a
t,h + O,h	1.05 + 0.33[be]	n/a	not chosen	n/a
t,h + t,h	not chosen	not chosen	0.25 + 0.25[b]	0.76 + 0.24[b]
O,t,h + O	not chosen	n/a	n/a	n/a
O,t,h + t	not chosen	not chosen	n/a	n/a
O,t,h + h	not chosen	n/a	5.28 + 0.05[ce]	n/a
O,t,h + O,t	2.75 + 1.20[a]	not chosen	n/a	n/a
O,t,h + O,h	5.12 + 3.01[ae]	n/a	not chosen	n/a
O,t,h + t,h	not chosen	not chosen	not chosen	1.64 + 0.87[be]
O,t,h + O,t,h	not chosen	3.98 + 2.61[ade]	not chosen	4.85 + 1.55[a]

Note: O = ones; t = tenths; h = hundredths; a = realistic; b = slightly unrealistic; c = very unrealistic; d = may adjust numbers before adding; e = may round total; n/a = not applicable

APPENDIX E: INSTRUCTIONS SCRIPT

You’ve all been given a sheet telling you about the study. Is there anything you want to know about it?

This study that you’re helping with will be useful in helping other kids learn maths. If you really don’t want to help, you can let me know at any time – you won’t get into any trouble.

There are 30 questions. Please try to answer all of them. I’d like to remind you that your teacher will not know your individual results, so this will not affect your report card in any way.

You can’t use a calculator or ruler but you can use any other way to work out the answers. If you like writing things down in a certain way, or like doing things in your head a certain way, that’s fine. But please show how you worked the answers out – if there were steps you made inside your head, write down what the steps were.

Once you have finished all the pages, turn them over face-down and raise your hand and I’ll come to collect them.

So,

1. Show your thinking for each question.
2. Raise your hand when you have finished or have a question – please don’t leave your seat.

[Note: what silent activity can students do when they’ve finished?]

Does anyone not know what to do?

You have 40 minutes to complete the test. Work silently please.

APPENDIX F: RESPONSE TYPE MARKING GUIDE

The *reported answer* is what is written next to the printed equals sign.
The *method answer* is what is written at the conclusion of a student's written method.
A *major error* is something that is substantially incorrect. A *minor error* results in an answer that is not fully correct, but is substantially correct in content.

Erased methods are analysed too if necessary to get a better understanding of a response.

*****
Codes starting with 0 have been marked incorrect.
Codes starting with a 1 have been marked correct.
Codes starting with a 2 may be correct or incorrect, depending on the scenario.
*****
Codes with 1 as the second digit involve mostly procedural issues.
Codes with 2 as the second digit involve place value.
Codes with 3 as the second digit involve rounding.
Codes with 4 as the second digit involve money notation.
*****
The third digit in the codes is nominal.
*****

Code	Description	Example	Result	Comment
ND	No answer at all		Not done	Not done at all
MI	Incorrect, with multiple errors	\$4.60 + \$1.00 = 809 4.03 + 4.06 8 09	Incorrect	*[identify then review]*
MX	A mix of correct and incorrect responses	\$6.00 + \$0.06 = 6.6	Incorrect	*[identify then review]*
MC	Correct, with multiple minor errors	\$6.00 + \$0.06 = 6.05 or 6.06	Correct	*[identify then review]*

110	Correct reported answer *and/or* method answer - can include minor errors that *do not* suggest conceptual errors	4.03 + 0.06 = 4.09 4.03 + 0.06 4.09 5.02 + 3.09 = 8.17 5.02 + 3.09 8.11 5.02 + 3.09 = 8.11 5.02 + 3.09 81.1 4.03 + 0.06 = 4.09 4.03 + 0.06 409	Correct	For CX problems, this also means correct use of dollar symbol. May involve: - incorrect transcribing of answer after calculation - missing decimal point in reported or method answer (not both)

010	Incorrect reported answer *and* method answer (if method is shown)	4.03 + 0.06 = 1.09 4.03 6+3=9 +0.06 6+4=10 1.09 3.98 + 2.61 = 1.37 3.98 2.61 1.37	Incorrect	Not covered by other types - may be procedural and/or conceptual Method may not be shown or is: - inappropriate for addition - dysfunctional; or - unintelligible
011	Incorrect reported answer *and* method answer (if shown) - functional method but <u>facts error</u>	3.98 + 2.61 = 7.49 13.98 +2.61 7.49 2.70 + 2.05 = 4.70 2.70 +2.05 4.70	Incorrect	Fluency error (basic facts) or conceptual error (re: adding 0) – also includes carry digit addition

012	Incorrect reported answer *and* method answer (if shown) - functional method but <u>mechanical error</u>	3.98 + 2.61 = 5.59 3.98 +2.61 5.59 3.98 + 2.61 = 10.19 ^{5}3.98 +2.61 10.19 4.85 + 1.55 = 15.40 ^{1}4.^{1}85 +1.55 15.40 4.03 + 0.06 = 8.09 4.03 +4.06 8.09	Incorrect	Procedural error (method) May involve: - extra, misread, absent or reversed carry digit - incorrect transcribing of question before calculation - misalignment of place value columns

012 cont.		4.85 + 1.55 = 86.55 85.00 + 1.55 86.55 1.64 + 0.87 = 1.51 4 + 7 = 11 60 + 80 = 140 140 + 11 = 151 198 + 80 = 998 198 +80 998 6.00 + 0.06 = 6.60 6.00 +0.60 6.60		

013	Missing decimal point - reported answer *and* method answer would otherwise be correct for amounts greater than 1 (CX and NCX questions) ***or*** less than 1 for NCX questions (cf. 030)	4.03 + 0.06 = 409 4.03 +0.06 409	Incorrect	Possible conceptual or transcribing error - conceptual accuracy (decimal fractions) **can't** be determined
120	Place value - "unnecessary" zeros removed from ones or hundredths place in reported answer and/or method answer	3.80 + 0.40 = 4.2 0.20 + 0.05 = .25	Correct	Possible conceptual links recognised or possible notation error
020	Place value - correct digits in the wrong places in reported answer ***and*** method answer. Also includes regrouping errors.	5.02 + 3.09 = 8.011 0.80 + 0.30 = 1.01 0.76 + 0.24 = 10.0 0.08 + 0.07 = 1.5 4.03 + 0.06 = 4.9	Incorrect	Possible conceptual (place value), procedural ***or*** transcribing error

020 cont.		3.98 + 2.61 = 5.24 3+2=5 9+6=15 8+1=9 24 0.99 + 0.70 = 0.169 0.99 +0.70 0.169 5.02 + 3.09 = 9.1 1 5.2 +3.9 9.1		

130	Rounding *after* addition - reported answer is different to method answer	4.03 + 0.06 = 4.10 4.03 +0.06 4.09	Correct	Procedural accuracy (basic facts) ***can*** be determined
030	Rounding *before* addition - reported answer is different to method answer	4.03 + 0.06 = 4.10 4.05 +0.05 4.10	Incorrect	Procedural accuracy (basic facts) ***can't*** be determined
140	Reported answer *and/or* method answer has no decimal format for whole dollars or amounts under a dollar but money notation is used correctly	\$0.25 + \$0.25 = 50c \$0.76 + \$0.24 = \$1	Correct	Possible notation error
141	Reported answer *and* method answer has no money notation but decimal format is used.	\$4.03 + \$0.06 = 4.09 \$0.18 + \$0.02 = 0.20	Correct	Notation error

240	Incorrect or contradictory money notation. Or money notation is missing *and* decimal fraction format is not used. Reported answer *and/or* method answer would otherwise be correct.	\$0.25 + \$0.25 = 0.50c \$4.03 + \$0.06 = \$4.09c \$0.25 + \$0.25 = 50	Incorrect *or* correct	Conceptual error ***or*** notation error

APPENDIX G: METHOD TYPE MARKING GUIDE

Code	Description	Example	Comment
ND	Not done	-	No answer is given.
0	No method shown or the question is restated horizontally	0.80 + 0.30 = 1.10 0.80 + 0.30 = 1.10 0.80+0.30 = 1.10 0.80 + 0.30 = 1.10 80+30 = 110 0.80 + 0.30 = 1.10 I added the numbers.	An answer is given but no method of calculation may be shown. If the question is rewritten it may or may not have money symbols and may or may not be in the decimal fraction format.

1	Standard written method (whether correctly performed or not)	0.99 + 0.70 = 1.69 $\begin{array}{r} \overset{1}{0}.99 \\ +0.70 \\ \hline 1.69 \end{array}$ 0.99 + 0.70 = 1.69 $\begin{array}{r} 99 \\ +70 \\ \hline 169 \end{array}$	An answer is given and the complete method is used though carry digits may be recorded in different ways or be absent. It may or may not have money symbols. Decimal points may be missing. “Unnecessary” zeros may be missing from decimal fraction part. May have counting marks.

2	Alternative written method. May involve a sub-addition that uses the standard written method. Method may be missing decimal point.	3.98 + 2.61 = 5.24 3.98 + 2.61 = 5.24 5 15 9 5.28 + 0.05 = 5.33 $\begin{array}{r} {}^{1}28 \\ +\ 5 \\ \hline 33 \end{array}$	Anything other than Type ND, Type 0 or Type 1. May have counting marks.

APPENDIX H: INFORMATION AND CONSENT FORMS

Locked Bag 1307 Launceston
Tasmania 7250 Australia
Phone 1800 061 512 Fax (03) 6324 3048
Email www@educ.utas.edu.au

UTAS

**PARENT INFORMATION SHEET
SOCIAL SCIENCE/ HUMANITIES
RESEARCH**

If you wish your child to take part in this study, please sign the attached consent form and return it to your child's teacher by XXXX.

INVESTIGATING STUDENTS' ADDITION OF DECIMAL FRACTIONS

Invitation

Your child is invited to participate in a research study into how accurate students are with addition of decimal fractions (e.g. 4.35).
The study is being conducted by:

Rosemary Callingham, Associate Professor, Faculty of Education
Allan Turton, Student Investigator

1. 'What is the purpose of this study?'

The purpose is to investigate whether accuracy with addition is greater when the numbers are used in a context, like working out the cost of two items.

2. 'Why has your child been invited to participate in this study?'

Your child is eligible to participate in this study because the researchers are interested in Year 4 and Year 5 students from schools that are similar in certain ways. These are schools where the results from the 2010 NAPLAN tests are close to what the Queensland average score was for mathematics. The researchers will not be looking at individual NAPLAN scores – the overall school score is only being used to identify suitable schools.

3. 'What does this study involve?'

The students in your child's class will be asked to complete short (30- to 40-minute), written worksheets of about 30 questions on mathematics. The worksheets will be completed in your child's usual classroom with their teacher present. The worksheets will be collected at the end and taken away for analysis. Any samples of your child's work will remain their intellectual property. The results from the worksheets will not be used to determine your child's grades or report cards in any way.

It is important that you understand that your child's involvement in this study is voluntary. While we would be pleased to have your child participate, we respect your right to decline. There will be no negative consequences for your child if you or they decide that they will not participate, and this will not affect how you or your child are treated by your child's teacher or school. If you or your child decide to withdraw from the study at any time, you or your child may do so without providing an explanation. All information will be treated in a confidential manner, and your name and your child's name will not be used in any publication arising out of the research. All of the research will be kept in a locked cabinet in the office of Rosemary Callingham.

4. Are there any possible benefits from participation in this study?
Your child's individual results will not be known by your child's teacher. However, the teacher will receive feedback about the class overall. The teacher may then decide to adjust their mathematics program which could benefit your child.

If we are able to take the findings of this small study and link them with a wider study, the result may be valuable information for others and it may lead to a better understanding of what Year 4 and Year 5 students are capable of in mathematics. It may mean that future instruction is more appropriate for their needs and abilities.

5. Are there any possible risks from participation in this study?
There are no specific risks anticipated with participation in this study. What your child will experience is similar to what they would usually experience at school.

6. What if I have questions about this research?
If you would like to discuss any aspect of this study please feel free to contact either Allan Turton on 0431 826 915 or Rosemary Callingham on 03 6324 3051. Either of us would be happy to discuss any aspect of the research with you. Once we have analysed the information we will be mailing / emailing your child's teacher a summary of our findings. You are welcome to contact us at that time to discuss any issue relating to the research study.

This study has been approved by the Tasmanian Social Science Human Research Ethics Committee. If you have concerns or complaints about the conduct of this study please contact the Executive Officer of the HREC (Tasmania) Network on (03) 6226 7479 or email human.ethics@utas.edu.au. The Executive Officer is the person nominated to receive complaints from research participants. You will need to quote H11797.

Thank you for taking the time to consider this study.
If you wish your child to take part in it, please sign the attached consent form and return it to your child's teacher by XXXX. This information sheet is for you to keep.

Locked Bag 1307 Launceston
Tasmania 7250 Australia
Phone 1800 061 512 Fax (03) 6324 3048
Email www@educ.utas.edu.au

UTAS

STUDENT INFORMATION SHEET
SOCIAL SCIENCE/ HUMANITIES
RESEARCH

INVESTIGATING STUDENTS' ADDITION OF DECIMAL FRACTIONS

Invitation
You are invited to help find out how many correct answers students can get when they add decimal fractions.

The study is being done by two researchers:
Rosemary Callingham, Associate Professor, Faculty of Education
Allan Turton, Student Investigator

1. 'What is the purpose of this study?'
We want to find out if writing maths questions in different ways makes it easier to answer them.

2. 'Why have you been asked to help?'
At the moment we only want to find out about Year 4 and Year 5 students. Your school has a lot in common with other schools in Queensland so that's why we chose your school.

3. 'What does this study involve?'
You will be asked to do some short maths worksheets. There are about 30 questions and it will take about 30 to 40 minutes to do them all. The worksheets will be collected at the end so we can study them. The results won't affect your grades or your report card.

It is important that you understand that you can choose to help or not help. While we would like to have your help, it is okay if you don't help. You won't get into trouble if you don't help and you can stop helping at any time without saying why. All you need to do is let Mr Turton know while he is in the classroom.

No-one but the researchers will know your results and your name will not be used if we write about this study. All of the research will be locked up in the office of Associate Professor Callingham.

4. Are there any good things that could happen?
Your teacher won't know your results but they will know how the class did overall. Your teacher may then work out if the class needs extra help with decimal fractions. This study could also help other people work out better ways of teaching maths to other students.

5. Are there any bad things that could happen?
We don't think there would be any bad things from helping. Answering the questions will be like what you usually do in class.

6. What if I have questions about this research?
If you have questions before, during or after doing the worksheets, please talk to Mr Turton while he is in the classroom. If you have questions about the study when Mr Turton is not in the classroom, please ask your teacher or parents to read the information sheets given to them to find out contact details.

Thank you for taking the time to think about this study.
You and your parents will be asked to sign a consent form.
This information sheet is for you to keep.

Locked Bag 1307 Launceston
Tasmania 7250 Australia
Phone 1800 061 512 Fax (03) 6324 3048
Email www@educ.utas.edu.au

UTAS

PRINCIPAL/TEACHER INFORMATION SHEET
SOCIAL SCIENCE/ HUMANITIES
RESEARCH

INVESTIGATING STUDENTS' ADDITION OF DECIMAL FRACTIONS

Invitation
Your students are invited to participate in a research study into how accurate students are with addition of decimal fractions (e.g. 4.35).

The study is being conducted by:
Rosemary Callingham, Associate Professor, Faculty of Education
Allan Turton, Student Investigator

1. 'What is the purpose of this study?'
The purpose is to investigate whether accuracy with addition is greater when the numbers are used in a context, like working out the cost of two items.

2. 'Why have your students been invited to participate in this study?'
Your students are eligible to participate in this study because the researchers are interested in Year 4 and Year 5 students from schools that are similar in certain ways. These are schools where the results from the 2010 NAPLAN tests are close to what the Queensland mean score was for Year 3 Numeracy. The researchers are not particularly interested in the school NAPLAN scores – the score is only being used to identify schools that will be representative of the wider Queensland school population.

3. 'What does this study involve?'
The students in your class will be asked to complete short (30- to 40-minute), written worksheets of about 30 questions on mathematics. The worksheets will be completed in your students' usual classroom with their teacher present. The student investigator will distribute the worksheets and answer student questions as required. It is not intended that the results of the worksheets will be used to determine your students' academic achievement in any way.

It is important that you understand that your and your students' involvement in this study is voluntary. While we would be pleased to have you and your students participate, we respect your and their right to decline. It is intended that there will be no negative consequences for your students if they decide that they will not participate, nor should it affect how your students are treated by you or other staff members. If you or your students' decide to withdraw from the study at any time, you or they may do so without providing an explanation. All information will be treated in a confidential manner, and your name, the school's name and your students' name will not be used in any publication arising out of the research. All of the research will be kept in a locked cabinet in the office of Rosemary Callingham.

4. Are there any possible benefits from participation in this study?
To maintain confidentiality, your students' individual results will only be known by the researchers. However, classroom teachers will receive feedback about the overall results of the class. Teachers may then decide to adjust their mathematics program which could benefit your students.

If we are able to take the findings of this small study and link them with a wider study, the result may be valuable information for others and it may lead to a better understanding of what Year 4 and Year 5 students are capable of in mathematics. It may mean that future instruction is more appropriate for their needs and abilities.

5. Are there any possible risks from participation in this study?
There are no specific risks anticipated with participation in this study. What your students will experience is similar to what they would usually experience at school.

6. What if I have questions about this research?
If you would like to discuss any aspect of this study please feel free to contact either Allan Turton on 0431 826 915 or Rosemary Callingham on 03 6324 3051. Either of us would be happy to discuss any aspect of the research with you. Once we have analysed the information we will be mailing / emailing you a summary of our findings. You are welcome to contact us at that time to discuss any issue relating to the research study.

This study has been approved by the Tasmanian Social Science Human Research Ethics Committee. If you have concerns or complaints about the conduct of this study please contact the Executive Officer of the HREC (Tasmania) Network on (03) 6226 7479 or email human.ethics@utas.edu.au. The Executive Officer is the person nominated to receive complaints from research participants. You will need to quote H11797.

Thank you for taking the time to consider this study.
If you wish your students to take part in it, please reply to Allan Turton at adturton@postoffice.utas.edu.au
This information sheet is for you to keep.

Locked Bag 1307 Launceston
Tasmania 7250 Australia
Phone 1800 061 512 Fax (03) 6324 3048
Email www@educ.utas.edu.au

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Please sign and return this form to your child's teacher by XXXX.

PARENT/GUARDIAN CONSENT FORM
Title of Project: INVESTIGATING STUDENTS' ADDITION OF DECIMAL FRACTIONS

1. I have read and understood the 'Information Sheet' for this project.
2. The nature and possible effects of the study have been explained to me.
3. I understand that the study involves doing mathematics worksheets that will take about 30 to 40 minutes to complete.
4. I understand that participation involves no risk that is greater than what my child usually would experience at school.
5. I understand that all research data will be securely stored on the University of Tasmania premises for five years and will be destroyed when no longer required.
6. Any questions that I have asked have been answered to my satisfaction.
7. I agree that research data gathered from my child for the study may be published provided that my child cannot be identified as a participant and that any samples of their work remain their intellectual property.
8. I understand that the researchers will maintain my child's identity confidential and that any information my child supplies to the researcher(s) will be used only for the purposes of the research.
9. I agree to my child participating in this investigation and understand that they may withdraw at any time without any negative effect, and if I or my child so wish, may request that any data they have supplied to date be withdrawn from the research.
10. I have discussed all the above information to my child and they have agreed to participate in the study.

Name of Parent/Guardian:

Signature: Date:

Name of Child (Participant):

Signature: Date:

Statement by Investigator

The parents/guardians of the participant have received the Information Sheet where my details have been provided so parents/guardians have the opportunity to contact me prior to consenting to participate in this project.

Name of Investigator: Allan Turton

Signature of Investigator: Date:

Locked Bag 1307 Launceston
Tasmania 7250 Australia
Phone 1800 061 512 Fax (03) 6324 3048
Email www@educ.utas.edu.au

UTAS

Please return to Allan Turton using the envelope provided.

PRINCIPAL CONSENT FORM
Title of Project: INVESTIGATING STUDENTS' ADDITION OF DECIMAL FRACTIONS

1. I have read and understood the 'Information Sheet' for this project.
2. The nature and possible effects of the study have been explained to me.
3. I understand that the study involves doing mathematics worksheets that will take about 30 to 40 minutes to complete.
4. I understand that participation involves no risk that is greater than what students usually would experience at school.
5. I understand that all research data will be securely stored on the University of Tasmania premises for five years and will be destroyed when no longer required.
6. Any questions that I have asked have been answered to my satisfaction.
7. I agree that research data gathered from students and staff for the study may be published provided that the school, staff and students cannot be identified.
8. I understand that the researchers will maintain the school's, staff and students' identities confidential and that any information supplied to the researcher(s) will be used only for the purposes of the research.
9. I agree to staff and students being invited to participate in this investigation and understand that I, the staff involved and the students (or their parents) may withdraw at any time without any negative effect, and if I, the staff involved and the students (or their parents) so wish, may request that any data that has been supplied to date be withdrawn from the research.

Name of Principal:

Name of School:

Signature: Date:

Statement by Investigator

I have explained the project & the implications of participation in it to this Principal and I believe that the consent is informed and that he/she understands the implications of participation.

Name of Investigator: Allan Turton

Signature of Investigator: Date:

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